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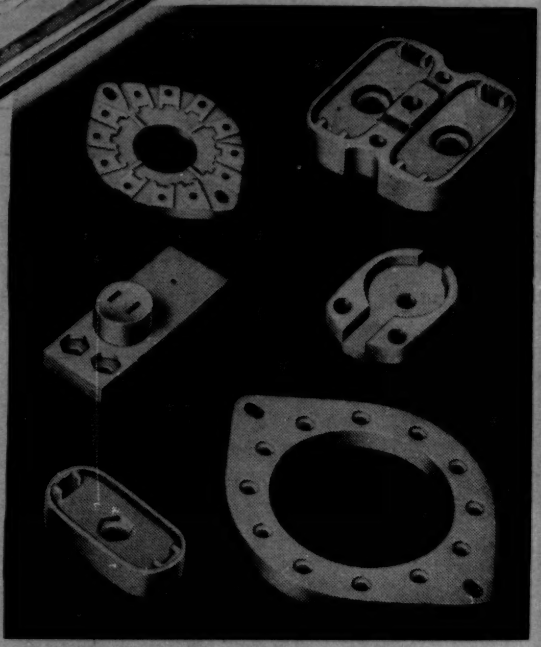

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WIRELESS ENGINEER

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EDITORIAL

The Use of Analogies

SOME years ago a student complained that whenever he asked a question about an electric circuit he was told to imagine a pipe full of water, etc. He said that he knew all about the pipe full of water but wanted to know about the electric circuit and he disapproved of being fobbed off with hydraulic analogies. We were reminded of this by some statements made by Professor Livens in a recent paper* on "The energy and mechanical relations of the magnetic field" in which he criticizes the use of magnetic and electric analogies as well as magnetic and mechanical analogies. He says, "The persistent adherence to a supposed analogy with the electrostatic case, which does not exist, and to frequently fanciful analogies of the stress-strain type, which have introduced more confusion than they have saved, seem to have blinded to the real solution those few authors who were able to see clearly that there are anomalies in the usual discussion, and the usual dimensional treatment which starts by assuming—again on the basis of some supposed stress-strain analogy—that the nature of the vectors of force and induction in the field must be different, has perhaps not helped in any way to clear the air for a more satisfactory treatment."

This raises some interesting and important points, especially for those engaged in teaching. Electrical engineering differs from mechanical engineering in that the student of the latter is dealing with concepts, such as force, mass, deflection, temperature, etc., with which he has been familiar since childhood, whereas the electrical student is introduced to a system of new and intangible concepts, such as potential difference, electromotive force, current, magnetic induction, etc. It is difficult to see how these can be explained without the use of analogies; the obvious way of explaining new and intangible concepts is by means of familiar and tangible concepts. It is, of course, essential to emphasize the danger of pushing the analogies too far; they should perhaps be regarded not as analogies but rather as parallel lines of thought. It is surely better for a student to have even a crude mental picture of what is happening in electric and magnetic phenomena than to have no mental picture at all, but to regard it all as a collection of words and symbols. The nomenclature of electromagnetism shows that it was largely based on mechanical analogies; the words current, pressure, tension, electromotive force, resistance, all suggest mechanical lines of thought, and the natural explanation of the unknown in terms of the known. We disagree with Professor Livens' contention that analogies of the stress-strain type have introduced more confusion than they have saved; on the contrary we think that the confusion that existed a few years ago with regard to B and H has been eliminated largely owing to the application of the stress-strain line of

*Phil. Mag. January 1945, Vol. XXXVI, p. 1.

thought, or, as some might prefer to call it, the cause and effect line of thought. When current passes round a coil it produces a magnetic field; when a pull is applied to a rod it produces an extension. The strain produced at any point in the material of the rod depends on the stress (i.e., the pull per unit area at the point) and on a property of the material. If one can assume that the stress is uniform over the cross-section it can be calculated by dividing the total pull by the cross-sectional area. Similarly the magnetic induction B produced at any point, say, in a mass of iron, must depend on some cause which we call the magnetizing force H at the point and some property of the material which we call the permeability. Instead of the total prime cause being distributed over a cross-sectional area as the pull is in the mechanical specimen undergoing a tension test, it is distributed over the length of a closed path. The prime cause in the magnetic case is the current or ampere-turns and the localized cause H is the current per unit length—neglecting such details as 4π.

Is Magnetizing Force a Force?

Just as in the special case of the stress being uniform over the cross-section, the localized cause is obtained by dividing the total cause by the area, so, in the special case of the uniformly wound toroid, the localized cause H is obtained by dividing the total cause, viz. the total current or ampere-turns, by the length. In both cases one has the total cause, the localized cause, and the localized effect depending on some property of the material, and we fail to see how such parallel lines of thought—or fanciful analogies as Professor Livens calls them—can introduce confusion or blind anyone to the real solution. We gather from Professor Livens' paper that there is some confusion due to a misunderstanding of the term "magnetic force" or "magnetizing force." When one refers to H as the magnetizing force one does not use the word "force" in its mechanical sense; the unit of H is not the dyne, although junior students sometimes make the mistake and give a field strength as so many dynes. It is true that H multiplied by a pole strength m gives a force in dynes, but this only shows that H itself cannot be a force in the mechanical sense. When we speak of a magnetic force or magnetizing force we mean the intensity of the magnetizing cause acting at a point. When one speaks of the force of circumstances or the force of an argument, one is using the word "force" to designate something that cannot be measured in dynes and, although somewhat far-fetched, these examples may serve as a warning against regarding H, by whatever name it may be called, as a mechanical force.

Cause (H) and Effect (B)

Now the mechanical force on a conductor carrying a current or on a moving electron depends on the magnetic induction or flux density B in which it is situated and not on the causative force H which produced the magnetic induction. This fact has led some people to suggest that B and not H should be called the magnetic or magnetizing force. In the paper to which we have referred, for example, Professor Livens says: "This last step in our thesis involves a complete inversion of the roles of B and H," and again "If this thesis is accepted, then we must prepare for a complete inversion of the whole of our traditional notions on this subject, that is, of course, if we admit that the claims of theoretical consistency must ultimately take precedence over those of traditional and almost universal and long standing practice." Apparently the idea is to refer to the magnetic induction B as the magnetizing force and to the magnetizing force H as the induction. We cannot imagine for a moment that Professor Livens expects his suggestion to be adopted.

If instead of an iron toroid linked by a current we have a dielectric toroid linked by a changing magnetic flux, the magnetic force or cause H is replaced by the electric force or cause $\mathcal{E}$ which depends only on the changing flux and the length of path and is independent of the material of the toroid. It is the causative force acting at any point and its effect will depend on the material of the toroid. I has been replaced by $d\phi/dt$ and μ by κ. Surely nothing but good can come from drawing the student's attention to these parallel lines of thought; it is one's duty to do so. In each case one can divide the effect into two parts, viz. that which would exist in the absence of the magnetic material or dielectric and that due to its presence, thus getting the two formulae

$$B = \mu_0 H + 4\pi J$$

and $4\pi D = \kappa_0 \mathfrak{E} + 4\pi P$

in which J is the intensity of magnetization, D the displacement and P the polarization of the dielectric. Also

$$\mu = \frac{B}{H} = \mu_0 + 4\pi J/H$$

and $$\kappa = \frac{4\pi D}{\mathfrak{E}} = \kappa_0 + 4\pi P/\mathfrak{E}$$

For the energy per cm^3 we have in one case $\mu H^2/8\pi$ and in the other $\kappa \mathfrak{E}^2 8\pi$.

In view of these formulae it is difficult to understand how anyone can complain of the "persistent adherence to a supposed analogy which does not exist." The parallelism is so striking that it should be emphasized when teaching the subject, but always with a warning that parallel lines of thought and symbolism do not indicate any physical likeness in the phenomena involved. G.W.O.H.

DIPOLE WITH UNBALANCED FEEDER*

By D. A. Bell, M.A., B.Sc.

IN the course of their paper on "Aerial Impedance Measurements" L. Essen and M. H. Oliver[1] mention that a half-wave dipole can be connected directly to a concentric feeder cable without any great impedance mis-match, but that they have not investigated the symmetry of the field.

The complementary observations have been made by the writer during the examination of directional aerials of the type commonly used for television reception, namely a vertical dipole associated with a single half-wave parasitic element. The aerial under examination was connected to a receiver, a low-power oscillator was set up about 10 wavelengths away, and a polar diagram of the aerial was measured by rotating the aerial supporting mast about its (vertical) axis and plotting received signal strength against the azimuthal angle of the aerial assembly. Two alternative methods of connecting the aerial to the receiver were available:

(i) Through screened and balanced twin cable and a balanced coupling-coil to the first tuned circuit of the receiver.

(ii) Through coaxial cable and a coupling coil having one side earthed and joined to the outer of the coaxial feeder. The coaxial cable was connected directly to the aerial, the central conductor to one limb of the dipole and the outer conductor to the other limb.

Preliminary tests with a simple dipole failed to reveal any substantial difference in signal strengths between the two feeder systems; but with the addition of a director adjusted to give maximum suppression in the backward direction, the balanced feeder gave a symmetrical polar diagram while the coaxial feeder gave an asymmetrical diagram. These are shown by the full line and dotted curves respectively in Fig. 1. This asymmetrical characteristic can produce a sharper minimum than would otherwise be available; but the exact shape of the characteristic is likely to vary with aerial site and feeder length, since it depends upon the "vertical-aerial pick-up" effect of the whole elevated system and upon the impedance of the outer conductor of the feeder.

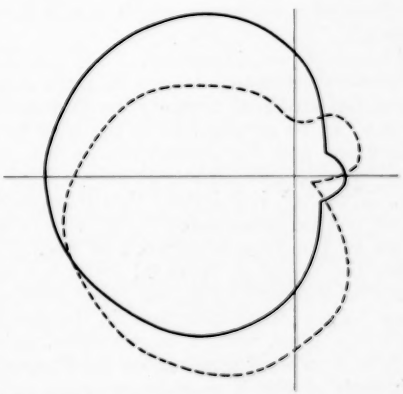

Fig. 1. *Polar diagrams of vertical dipole and director with balanced (solid line) and unbalanced (dotted line) feeders.*

A balanced aerial should preferably be connected to a coaxial feeder through a balancing device, of which the simplest form

*MS. accepted by the Editor, July 1946.

is the quarter-wave " sleeve " as shown by Essen and Oliver in their Fig. 8, and the equivalent circuit shown in Fig. 2 gives an idea of the behaviour of a centre-fed half-wave dipole connected in this way. The symmetrical dipole is represented by a source of e.m.f. E_1 at the centre of a resistance of 76 ohms total value, and the terminals A, B of the dipole are connected to a feeder which is assumed to be of 76 ohms characteristic

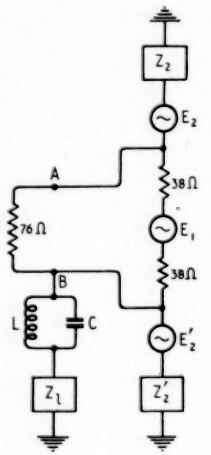

Fig. 2. *Equivalent circuit of dipole aerial.*

impedance and non-reflectively terminated : so far it makes no difference whether the feeder consists of balanced-twin or coaxial cable, and no " earth " has been introduced. Practical dipoles, however, are not mounted in free space but usually near an " earth," whether this be the surface of the ground, the body of an aircraft, or the general structure of a tower on which the aerial is mounted.

Now it is well-known that an elevated conductor connected to earth by a wire constitutes an aerial system : an alternating current can be established in the wire by a radio wave, and this current can be detected by a suitable instrument as shown in Fig. 3(a). Moreover, it is known that this phenomenon can be represented by an equivalent circuit of the type illustrated in Fig. 3(b), where the aerial is replaced by a voltage generator E in series with an " aerial impedance " Z_{ae}. Returning now to the dipole, when associated with a feeder cable going down to a receiver situated on the " earth " each limb of this is equivalent to an aerial of the type of Fig. 3(a), and according to the equivalent circuit of Fig. 3(b) this fact is indicated in Fig. 2 by adding the generators E_2 and E'_2 each in series with an appropriate impedance Z_2, Z'_2. Since the two limbs are similarly situated above the earth, the two generators will be in the same phase, and so long as the load connected between A and B is balanced to earth, they will introduce no net difference of potential between these terminals. But if, for example, terminal B were directly earthed, the generator E_2 would be connected in series with Z_2 and the load, and would cause a voltage to appear across the load ; i.e., across the receiver input circuit.

In most cases the outer of the concentric feeder is earthed at the receiving end of the feeder, not at the aerial end, and the impedance to earth presented at B is then a function of feeder length : for odd multiples of $\lambda/4$ it is very high and for even multiples very low if the transmission line made up of cable outer and earth is of low loss. In practice, there is likely to be considerable loss due to the waterproof covering over the outer of the cable and to miscellaneous objects between cable and earth ; but it is perhaps worth noting also that Schelkunoff[2] has given figures for the input impedance of an infinitely long conductor in free space, and for television feeder and frequencies as used by the writer it is of the order of 500 ohms. Since losses will damp out any resonance effects, it seems probable that the impedance to earth presented by the length of outer conductor of the coaxial feeder would be of the order of 500 ohms.

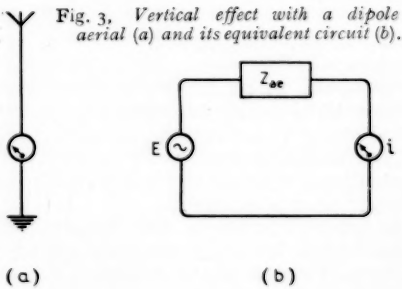

Fig. 3. *Vertical effect with a dipole aerial (a) and its equivalent circuit (b).*

This is the impedance represented by Z_1 in Fig. 2, and two deductions can be made from the magnitude assigned to it : (i) So long as the load impedance between A and B is small compared with 500 ohms, the generator E_2 is likely to produce only a secondary effect, and (ii) any impedance (such as that represented by L, C in Fig. 2) which may be added in order to reduce this effect must be appreciably greater than 500 ohms. The resonant-line analogue of the parallel-

resonant circuit L, C of Fig. 2 is a short-circuited quarter-wave line, or in other words the quarter-wave "sleeve" on the outer of the concentric cable to which reference has already been made, and which is illustrated in more detail in Fig. 4. As an alternative to the general explanation that a short-circuited quarter-wave line presents a high impedance at the open end, the mode of operation of the sleeve may be interpreted as follows. Assuming the sleeve to be a perfect conductor, it will act as a screen so that any current in the cable produces no external field; and this implies that for any current i_1 down the conductor 1 (the outer of the coaxial cable), there must be an equal and opposite current $-i_1$ along the inside of the sleeve, and these currents will balance at the closed end of the quarter-wave so as to leave no net current to flow down the remainder of the feeder. The resonance of the quarter-wave is necessary to minimize the unbalancing of the dipole due to attaching this impedance to one limb of it but not to the other. (There are more elaborate balance-to-unbalance transformers which maintain a higher degree of symmetry but also function as "stubs" in parallel with the balanced circuit.)

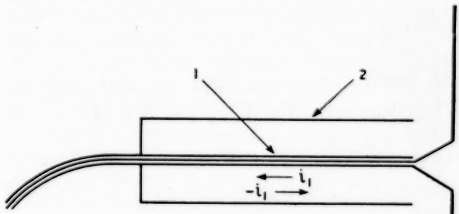

Fig. 4. *Quarter-wave balancing sleeve.*

This note is based on work carried out in the Research Laboratories of A. C. Cossor Ltd., and acknowledgments are due to Mr. L. H. Bedford, Director of Research, and to other members of the Research Staff with whom these questions have been discussed.

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1 *Wireless Engineer*, December 1945, Vol. XXII, p. 587.

2 "Theory of Antennas of Arbitrary Size and Shape," by S. A. Schelkunoff, *Proc. Inst. Rad. Engrs.*, September 1941, Vol. 29, p. 493.

THEORY OF THE EQUIVALENT DIODE*

By G. B. Walker

(The Mullard Radio Valve Co., Ltd.)

SUMMARY.—The theory of the equivalent diode is discussed and a new method, based on electrostatic considerations, is suggested whereby the equivalent diode can be uniquely determined.

Introduction

THE "equivalent" diode is a concept which has always lacked a precise definition, and like some other concepts in electronic engineering has taken on a different colouring to suit the taste of the user. The importance of the concept lies in the fact that it makes a rough calculation of the current passed by a triode possible, but there can be no doubt that if the physics of the triode were more simple the concept would never have arisen. One might be led to believe, therefore, that the equivalent diode is no more than a useful device, essentially arbitrary and having no real existence, but we feel that the logic of the question still merits consideration.

* MS accepted by the Editor, June 1946.

The supposition that a diode can be constructed which will have the same current-voltage relationship as a given triode has a practical basis in the discovery by Van der Bijl[1] in 1913 that for a limited range of applied voltages the current passed by a triode obeys the empirical law—

$$I = k(V_a + \mu V_g + \epsilon)^\beta \quad .. \quad .. \quad (1)$$

k, μ, ϵ and β being constants.

The form of this law, in which the applied voltages are combined in a linear term, together with the experimental fact that β has a value in the neighbourhood of 3/2 indicates that over the limited voltage range a triode behaves as a diode. This is the only justification for an attempt to form the conception of an equivalent diode but we must appeal to theory for aid in advancing

the matter. It is at this point that a divergence of opinion occurs.

The Electrostatic Approach

The favourite approach to the question, and in the writer's opinion the most consistent, as will be shown later, is to postulate that a diode is equivalent to a triode if, when the cathode is cold and no current is flowing, the off-cathode electrostatic field strength is the same in both valves. Let us examine this requirement for the case of a triode having planar electrodes.

The average value of the off-cathode field strength in the absence of space current can be written—

$$F = \frac{V_a + \mu V_g}{a + \mu g} \quad .. \quad .. \quad .. \quad (2)$$

where g is the spacing between cathode and grid and a is the spacing between the cathode and anode. μ is the amplification factor of the valve calculated on electrostatic grounds and depending only on the geometry of the valve.

For the equivalent diode we can write

$$F = \frac{V_d}{d} \quad .. \quad .. \quad .. \quad .. \quad (3)$$

where d is the spacing between cathode and anode.

Thus

$$\frac{V_d}{d} = \frac{V_a + \mu V_g}{a + \mu g}$$

This equation is insufficient to determine both V_d and d, so let us write

$$V_d = \alpha(V_a + \mu V_g)$$

and

$$d = \alpha(a + \mu g)$$

where α is some, so far undetermined, dimensionless constant.

Now the current passed by the diode is

$$I = 2.34 \times 10^{-6} \frac{V_d^{\frac{3}{2}}}{d^2}$$

(omitting the term ϵ in (1) which takes contact-potential differences into account), and so for the triode we have

$$I = 2.34 \times 10^{-6} \frac{(V_a + \mu V_g)^{\frac{3}{2}}}{\sqrt{\alpha}\,[a + \mu g]^2}$$

The difficulty now is to evaluate α or, which is the same thing, find the proper spacing in the equivalent diode. Many plausible though divergent arguments have been given which need not be discussed here.

Most writers give it the value $\frac{1}{\mu}$, (c.f. Dow[2] and Benham[3]), the latter coming to this decision after considering no less than six alternatives. There can be no doubt that in most cases the value should be in the neighbourhood of $\frac{1}{\mu}$ as current measurements on a triode will confirm, but let us examine what happens when μ tends to zero. In the limit this is equivalent to the removal of the grid from the valve and we can deduce, therefore, that the current passed should be

$$I = 2.34 \times 10^{-6} \frac{V_a^{\frac{3}{2}}}{a^2}$$

whereas according to (7) if $\alpha = \frac{1}{\mu}$ and μ tends to zero I vanishes!

The Current Approach

To obviate the difficulty in determining α and at the same time to pay greater attention to the behaviour of space charge in the valve some writers have rejected the electrostatic approach. We shall now examine an ingenious method given by Fremlin[4] which illustrates the essential features of the current approach.

Fremlin accepts the expression

$$I = k\,(V_a + \mu V_g)^{\frac{3}{2}} \quad .. \quad .. \quad (9)$$

and evaluates k in the following way.

If the grid of the triode is removed the anode current is,

$$I = 2.34 \times 10^{-6} \frac{V_a^{\frac{3}{2}}}{a^2}$$

or

$$V_a = \frac{I^{\frac{2}{3}}}{(2.34 \times 10^{-6})^{\frac{2}{3}}}\, a^{\frac{4}{3}} \quad .. \quad (10)$$

The potential at the plane where the grid was, is

$$V_g = \frac{I^{\frac{2}{3}}}{(2.34 \times 10^{-6})^{\frac{2}{3}}}\, g^{\frac{4}{3}} \quad .. \quad .. \quad (11)$$

and so if the grid is replaced and maintained at that potential no change should occur in the current passed by the valve. Thus, by substituting (10) and (11) in (9) he obtains

$$I = k\left[\frac{I^{\frac{2}{3}}\, a^{\frac{4}{3}}}{(2.34 \times 10^{-6})^{\frac{2}{3}}} + \mu \frac{I^{\frac{2}{3}}\, g^{\frac{4}{3}}}{(2.34 \times 10^{-6})^{\frac{2}{3}}}\right]^{\frac{3}{2}}$$

$$\text{or } k = \frac{2.34 \times 10^{-6}}{[a^{\frac{4}{3}} + \mu g^{\frac{4}{3}}]^{\frac{3}{2}}}$$

$$\text{and so } I = 2.34 \times 10^{-6} \frac{(V_a + \mu V_g)^{\frac{3}{2}}}{[a^{\frac{4}{3}} + \mu g^{\frac{4}{3}}]^{\frac{3}{2}}} \quad .. \quad (12)$$

Apparently the equivalent diode has disappeared from the argument, but if we compare expressions (12) and (7) we see that in Fremlin's treatment α is given the value $\frac{[a^{\frac{4}{3}} + \mu g^{\frac{4}{3}}]^3}{[a + \mu g]^4}$. In this way it is possible to define an equivalent diode.

Critical Discussion

Let us examine the implications in the use of the $\frac{3}{2}$-power voltage law.

It has been shown by Langmuir and Compton[5] that the current passed by a diode, irrespective of the shape of the electrodes, is proportional to the $\frac{3}{2}$ power of the anode voltage on the one condition that electrons leave the cathode with no initial velocity. In the proof the fact emerges that the potential at any point in the field is proportional to the anode voltage. It cannot be disputed, therefore, that Fremlin's formula for the current passed by an ideal triode (ideal in that there is no emission velocity) is correct, but only for values of grid and anode voltage such that $\frac{V_g}{V_a} = \left[\frac{g}{a}\right]^{\frac{3}{2}}$.

For the law (1) to have any practical value it is essential that V_g and V_a can be varied independently, or to put the matter otherwise, it is essential that μ is constant. Now in a valve, μ is approximately constant only under two conditions :—

(1) that space charge is negligible in the region between the grid and the anode, and

(2) that the disturbance of the electric field by the wires of the grid does not extend so far as to cause field irregularities at the cathode.

We can assume that condition (2) is satisfied in Fremlin's case but condition (1) is certainly not. In fact it is necessary for him to suppose that space charge is "saturated" between the grid and anode. This is undoubtedly a weakness in his position. From a practical point of view the part of the triode characteristic which most nearly conforms to Van der Bijl's empirical law corresponds to a range of grid voltage in the neighbourhood of zero. In this range μ is hardly affected by space charge and here only is it legitimate to combine the grid and anode voltages in a linear term.

A New Suggestion

To summarize the position at which we have arrived, we may say that law (1) applies only when it is impossible to tell whether the valve is a diode or a triode by observing the field at the cathode and when space charge has a negligible influence on the field beyond the grid. Now in the electrostatic approach we postulated that in the absence of space charge the electric field strength at the cathode must be identical in the triode and equivalent diode. Let us postulate further that the variation in that field strength with a change in cathode voltage is also the same for both. This is not a new fact but is a fuller statement of identity.

In the general case, the field a the triode cathode can be written as a linear function of the applied voltages as follows :—

$$F = A[(V_a - V_k) + \mu(V_g - V_k)]$$

and for the diode

$$F' = A'(V_d - V_k)$$

where A, A' and μ depend on the valve dimensions only. In addition to the condition

$$F = F'$$

we have also $\frac{\partial F}{\partial V_k} = \frac{\partial F'}{\partial V_k}$

thus $A[(V_a - V_k) + \mu(V_g - V_k)] = A'(V_d - V_k)$

and $A(1 + \mu) = A'$

Writing $V_k = 0$ we obtain

$$V_d = [V_a + \mu V_g]/(1 + \mu)$$

or $\alpha = 1/(1 + \mu)$

and the corresponding expression for the current with planar electrodes becomes

$$I = 2.34 \times 10^{-6} \frac{\sqrt{1 + \mu}\,[V_a + \mu V_g]^3}{[\alpha + \mu g]^2}$$

Like Fremlin's result this expression is correct when μ tends to zero or infinity. In practice μ is rarely less than 3 and so the departure of $\sqrt{\frac{\mu}{1 + \mu}}$ from unity is small.

One point in favour is that the 'electrostatic' μ is used and the formula is restricted to the range in which the true μ and the electrostatic μ coincide.

The principal difficulty in obtaining experimental verification of these formulae is caused by the neglect of emission velocity. It is to be noted, however, that the equivalent diode as deduced by the last-mentioned method is unique whatever the emission velocity may be.

Acknowledgment

The author wishes to express his indebtedness to the Management of the Mullard Radio Valve Company Limited for permission to publish this work.

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DESIGN OF CONSTANT IMPEDANCE EQUALIZERS*

By A. W. J. Edwards, Dipl. Ing., A.M.I.E.E.

(Formerly of the Engineer-in-Chief's School, P O. Eng. Dept.)

SUMMARY.—Some useful properties of inverse networks (as used in line equalization) are deduced and applied to the development of simple practical design procedures involving no calculations when suitable test equipment is available.

1. Introduction

THE method suggested in this article applies to all bridged-T, ⌐ and L networks fulfilling the condition that $a \times b = Z_0^2$ where a and b are the impedances designated as such in Figs. 1(a), (b), and (c), and Z_0 is the iterative impedance of the respective network. So far as the bridged-T network is concerned a modified form is sometimes employed where the branch impedances shown as Z_0 in Fig. 1(a) appear modified by the factor $1/c$. A network of this kind [shown in Fig. 1(d)] cannot be treated *directly* by the method to be developed. It will, however, be shown in the appendix that this network is obtained from the one of Fig. 1(a), for which $c = 1$, by a simple star-mesh transformation.

Lattice networks can be treated by a modified version of the suggested design method, which will also be discussed.

* MS. accepted by the Editor, May 1946.

2. Theoretical Basis of Design

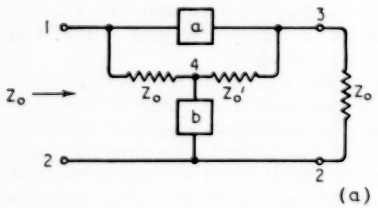
The theoretical basis of the design can be represented in the form of three propositions, for which proofs will be given under 2.2.

2.1. *The three principles of the design*

I(a). A bridged-T network with $c = 1$ can be converted into an ⌐ network of the same iterative transfer coefficient and the same iterative impedance by short circuiting the Z_0 arm next to the output terminals of the network.

I(b). A bridged-T network with $c = 1$ can be converted into an L network of the same iterative transfer coefficient and the same iterative impedance by disconnecting the Z_0 arm next to the output terminals of the network.

These two rules obviate the necessity of treating the three networks separately. The ⌐ and the L network are simply viewed as derivatives of the bridged-T network.



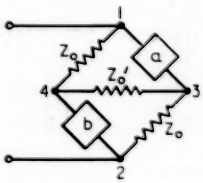
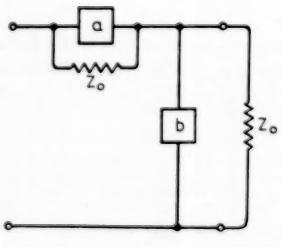
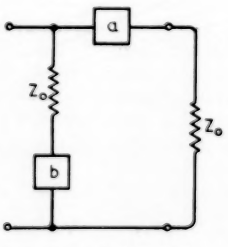
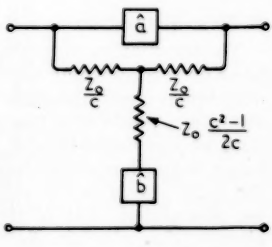


Fig. 1. *Bridged-T and the equivalent bridge networks are shown at (a), with ⌐ and L networks at (b) and (c). A modified form of (a) appears at (d).*







II(a). If the shunt arm of a bridged-T network with $c = 1$ is connected in series with an impedance equal to the iterative impedance of the network, the natural logarithm of the vector ratio of the voltage across both, to the voltage across the shunt arm is equal to the iterative transfer coefficient of the whole network.

The following is a dualistic restatement of the above and is only given for the sake of completeness.

If the shunt arm of a bridged-T network with $c = 1$ is connected in parallel with an impedance equal to the iterative impedance of the network, the natural logarithm of the vector ratio of the total current to the current in the iterative impedance is equal to the iterative transfer coefficient of the whole network.

II(b). If the significant series arm [i.e., "a" in Fig. 1(a)] of a bridged-T network with $c = 1$ is connected in series with an impedance, equal to the iterative impedance of the network, the natural logarithm of the vector ratio of the voltage across both to the voltage across the iterative impedance is equal to the iterative transfer coefficient of the whole network.

Restating the above:

If the significant series arm of a bridged-T network with $c = 1$ is connected in parallel with an impedance equal to the iterative impedance of the network the natural logarithm of the vector ratio of the total current to the current in the series arm is equal to the iterative transfer coefficient of the whole network.

The first forms of statements II(a) and II(b) are the basis of the new method.

III. A bridged-T network with $c = 1$ can be converted into a lattice of the same iterative impedance and iterative transfer coefficient if the impedance of its shunt arm is doubled, placed in series with an impedance equal to the iterative impedance and used as one of the four branches of the lattice; the branch adjacent to it is then found by halving the impedance of the significant series arm of the bridged-T network and placing it in parallel with an impedance equal to the iterative impedance.

This rule enables the method to be extended to lattice networks.

2.2. *Proofs for the three principles employed. I(a) and I(b)*

The bridged-T network of Fig. 1(a) can be redrawn as a bridge in the manner shown. The condition for balance of this bridge is $a \times b = Z_0^2$; if this condition is fulfilled the current and voltage distribution in the remainder of the network will be unaffected by the value of the "galvo-arm." If the latter is short-circuited Fig. 1(b) results, if open circuited Fig. 1(c). The only purpose of the impedance Z_0' [terminals 4 and 3 of Fig. 1(a)] is to make the network symmetrical.

It remains to prove that the condition $a \times b = Z_0^2$ furnishes iterative networks which can be used as equalizers.

Consider Fig. 1(a) and assume terminals 4 and 3 to be open-circuited. This is permissible in the light of the above. The input impedance between terminals 1 and 2 will be

$$Z_{IN} = \frac{(a + Z_0)(b + Z_0)}{a + b + 2Z_0} = \frac{ab + Z_0 b + Z_0 a + Z_0^2}{a + b + 2Z_0}$$

but $ab = Z_0^2$;

$$Z_{IN} = \frac{Z_0^2 + Z_0 b + Z_0 a + Z_0^2}{2Z_0 + a + b} = \frac{Z_0(2Z_0 + a + b)}{2Z_0 + a + b} = Z_0$$

This proves that the input impedance of all three networks is the same as the load impedance if $a \times b = Z_0^2$; they are therefore suitable as equalizers. Normally Z_0 will be independent of frequency and purely resistive. The reasoning employed is, however, still valid if Z_0 is a function of frequency and has an angle, so long as the product $a \times b$ is made equal to Z_0^2 at all frequencies.

II(a) and II(b)

In view of what has been said before, it is clear that the iterative transfer coefficient is the same for the three networks. It can easily be found in terms of Z_0 and a or Z_0 and b if, for instance, terminals 4 and 3 Fig. 1(a) are considered open-circuited. The well-known expressions for a and b follow readily and are given here for reference:

$$b = \frac{Z_0}{e^\theta - 1}; \quad a = Z_0(e^\theta - 1)$$

where θ is the iterative transfer coefficient of the networks.

If now branch b is placed in series with Z_0, as shown in Fig. 2(a), the ratio of the

applied voltage to the voltage across b is given by :—

$$\frac{V_1}{V_2} = \frac{Z_0 + b}{b} = \left(Z_0 + \frac{Z_0}{\epsilon^\theta - 1}\right)\left(\frac{\epsilon^\theta - 1}{Z_0}\right)$$
$$= \epsilon^\theta = \epsilon^{\alpha} \angle \beta$$

Similarly for branch a, Fig. 2(b). The ratio of the applied voltage, to the voltage across Z_0 is given by

$$\frac{V'}{V''} = \frac{Z_0 + a}{Z_0} = \frac{Z_0 + Z_0(\epsilon^\theta - 1)}{Z_0}$$
$$= \epsilon^\theta = \epsilon^{\alpha} \angle \beta$$

where α is the attenuation of the complete network in nepers and β its phase-shift in radians.

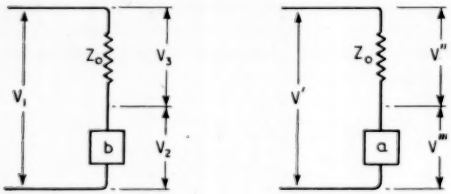


Fig. 2. *Basic circuits used in determining the networks " a " and " b."*

III

The well-known expressions for the two arms of a lattice network are

$Z_1 = Z_0 \tanh \frac{\theta}{2}$ and $Z_2 = Z_0 \coth \frac{\theta}{2}$ where Z_1 and Z_2 are the impedances so designated in Fig. 3.

This can be rewritten as

$$Z_1 = Z_0 \frac{\epsilon^\theta - 1}{\epsilon^\theta + 1} \text{ and } Z_2 = Z_0 \frac{\epsilon^\theta + 1}{\epsilon^\theta - 1}$$

It follows from these expressions that Z_1 can be considered as consisting of $Z_0(\epsilon^\theta - 1)$ in parallel with a similar impedance and both in parallel with Z_0.

The proof is simple :

$$Z_0(\epsilon^\theta - 1) \text{ in parallel with } Z_0(\epsilon^\theta - 1)$$
$$= \frac{Z_0(\epsilon^\theta - 1)}{2}$$

$$\frac{Z_0(\epsilon^\theta - 1)}{2} \text{ in parallel with } Z_0$$
$$= \frac{Z_0^2(\epsilon^\theta - 1)}{Z_0(\epsilon^\theta - 1) + 2Z_0} = Z_0 \frac{\epsilon^\theta - 1}{\epsilon^\theta + 1};$$

Similarly the branch Z_2 can be considered as consisting of $Z_0 \frac{1}{\epsilon^\theta - 1}$ in series with a similar impedance and both in series with Z_0.

The significant elements of this type of network are therefore equal to $Z_0(\epsilon^\theta - 1)$ and $Z_0 \frac{1}{\epsilon^\theta - 1}$ respectively and can be found by the method discussed (see section 3.3).

3. Practical Design Procedure

3.1. *To meet given attenuation requirements*

Considering Figs. 2(a) and (b) and applying Principles II(a) and (b) we get

$$\frac{V_1}{V_2} = \epsilon^\theta \text{ and } \frac{V'}{V''} = \epsilon^\theta ;$$

if only the moduli of the two voltages are of interest we can write :

$$\frac{|V_1|}{|V_2|} = \epsilon^\alpha \text{ and } \frac{|V'|}{|V''|} = \epsilon^\alpha$$

If actual measurements are carried out the voltmeter used will, of course, ignore the angles. If instead of a voltmeter a transmission measuring set if employed the readings corresponding to V_1 and V_2 will be $M = \log_e \frac{|V_1|}{|V_R|}$ and $m = \log_e \frac{|V_2|}{|V_R|}$ respectively, if V_R is the reference voltage and the set is calibrated in nepers.

The difference between the two readings is then :

$$M - m = \log_e \frac{|V_1|}{|V_R|} - \log \frac{|V_2|}{|V_R|}$$
$$= \log_e \frac{|V_1|}{|V_2|} = \log_e \epsilon^\alpha = \alpha$$

In other words the difference between the two readings gives the attenuation of the bridged-T or of the L network of which b forms a part, in the units in which the set is calibrated. Similar considerations hold for branch a and the voltages V' and V'''.

This enables us to find the component values of, say, branch b experimentally. All

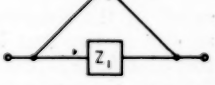


Fig. 3. *Basic lattice network.*

that is necessary is to adjust the components until $M - m$ is equal to the desired attenuation of the complete network at all frequencies of importance. The branch a is then found by carrying out a similar adjustment, or, for preference, simply by making it inverse to b with respect to R_0, following the well-known rules for networks which are inverse with respect to a constant resistance.

As an example, the detailed procedure for the adjustment of an equalizer suitable for a lump-loaded line is discussed in Appendix II.

3.2. *To equalize a given line*

If the line for which the equalizer is intended is accessible, a more elegant way of carrying out these adjustments is possible and was suggested to the author by Mr. H. J. Orchard, of the Engineer-in-Chief's School, P.O. Eng. Dept.

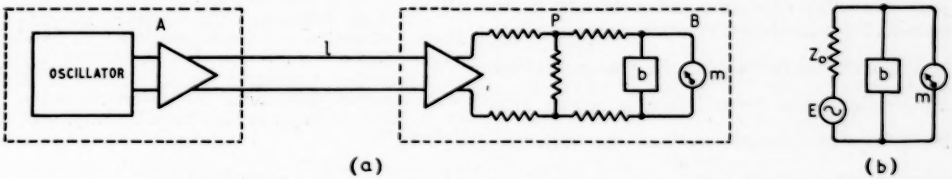


Fig. 4. *Circuit in (a) shows method of determining "b" arm of equalizer for line "l" which is available for test. Equivalent electrical circuit is shown in (b).*

The circuit of Fig. 4(a) is set up, where l is the line for which the equalizer is to be adjusted and P is an attenuator of say, 10 db, to mask any deviations of the output impedance of the amplifier from its design impedance.

Provided the design impedances of the attenuator and associated amplifier are equal to the iterative impedance of the equalizer, the equivalent circuit of this arrangement, shown in Fig. 4(b), is identical with the circuit previously used.

Inspection of Figs. 4(a) and (b) shows that E [the equivalent e.m.f. in Fig. 4(b)] when expressed in db will vary with frequency by exactly the same amount as that by which A (the line attenuation) varies with frequency, if a constant input is applied to the line and the amplifier gain is assumed to be flat with frequency.

Since E in db corresponds to M in section 3.1 and in Appendix II, it follows from the results obtained in this section that the components in b should be adjusted so that m [the reading of a db meter, Fig. 4(a)] is the same at all frequencies and equal to the value obtained at the highest frequency.

3.3. *Lattice networks*

Principle III points the way to an experimental design for lattice networks. One could, of course, simply find the arms a and b of a bridged-T network having the same iterative parameters as the desired lattice and convert to the lattice afterwards.

In practice it is more convenient to employ the circuit of Fig. 5 which enables us to find the arms of the lattice directly. It has been shown that the arm Z_2 of the lattice of Fig. 3 $\left(\text{equal to } Z_0 \coth \frac{\theta}{2}\right)$ can be considered as consisting of an impedance equal to Z_0 in series with an impedance $q = 2 \times Z_0 \frac{1}{\epsilon^{\theta} - 1}$.

Applying the basic idea of Proposition II to this circuit it follows that $\frac{|V^{x}|}{|V^{xx}|} = \epsilon^{\alpha}$, where α is the attenuation of the complete lattice, V^{x} is the voltage applied to this experimental circuit and V^{xx}, the voltage across q, is the "significant" part of the lattice arm. Therefore q can be found experimentally in the usual manner and is then placed in series with an impedance equal to Z_0 to furnish the complete arm of the desired lattice. The other arm of the lattice can be found in an analogous manner or better by reciprocation.

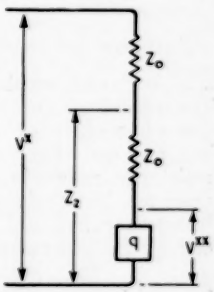


Fig. 5. *Circuit used in finding the arms of a lattice network.*

3.4. *Inverse networks not of constant-resistance.*

Networks of the bridged-T and L type whose impedance is not constant with frequency can be designed by adapting the above method.

It will be necessary to design a two-terminal network first, whose impedance-frequency response follows the desired law. This network is then used in place of R_0 either in actual experiment or for theoretical investigations. Let the impedance of this network $= Z_0(\omega)$. The branches a and b will

now be inverse only in the wider meaning of the word; i.e., the product $a \times b$ will still be equal to Z_0^2, but as Z_0^2 is not constant with frequency and may have an angle, reciprocation cannot be carried out by following the simple rules which apply to constant-resistance networks. Two ways out of the difficulty offer themselves.

One can either find both networks a and b separately by means of a method similar to the one discussed, or else find one network first and determine the other by means of the bridge circuit of Fig. 6.

In the first case one network, say b, is obtained by finding a suitable configuration and its component values such that $\frac{|V_1|}{|V_2|}$ [Fig. 2(a)] follows the desired law, a record also being kept of the ratio $\frac{|V_1|}{|V_3|}$ at the different frequencies.

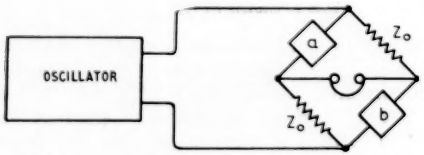


Fig. 6. *Bridge circuit used in the determination of networks which are not of constant-resistance type.*

But now $\frac{V_1}{V_2}$ must be equal to $\frac{V'}{V''}$ [Fig. 2(b)] both with respect to modulus and angle. The angles of the voltage ratios represent the phase-shift of the ultimate network. We are no longer dealing with constant-resistance networks, where the angle of a need not be considered, as it automatically becomes equal and opposite to the angle of b by the process of reciprocation. Having kept a record of $\frac{|V_1|}{|V_3|}$ one could now find a configuration for a (which will not simply be inverse to b in the narrower sense of the word) and adjust its components until $\frac{|V'|}{|V''|} = \frac{|V_1|}{|V_2|}$ and $\frac{|V'|}{|V'''|} = \frac{|V_1|}{|V_3|}$ at all frequencies of consequence, this will make the angles of the two networks equal, but they may still have the wrong sign, which would have to be decided from inspection.

This method is cumbersome and the use of the bridge circuit of Fig. 6 recommends itself. Once one network, say b, has been found by paying attention to the modulus of $\frac{V_1}{V_2}$ only, the network is inserted in the bridge. The configuration and component values of a are then found by adjusting until the bridge is balanced over the prescribed range; $a \times b$ will then be equal to Z_0^2 at those frequencies.

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"Constant Impedance Equalizers: Simplified Method of Design and Standardisation," F. Pyrah, *P.O. elect. Engrs J.*, Vol. 32, p. 204, October 1939.

"Distortion Correction in Electrical Circuits with Constant Resistance Recurrent Networks." O. J. Zobel, *Bell System tec. Journal*, Vol. 7, No. 3, July 1928.

APPENDIX I

1. *"Derived" bridged-T networks*

The completed branch b of a bridged-T network ($c = 1$) will normally contain a series resistance R which, in conjunction with an inverse shunt resistance S of branch a determines the basic loss of the network. The *remainders* of the branches a and b are shown as $\hat{a}$ and $\hat{b}$ [see Fig. 7(a).]

In order to save one resistor a transformation from mesh to star can be carried out according to Fig. 7(b), where k, k' and p are the branches of a star replacing the mesh consisting of S, R_0 and R_0'. Resistor p is of course combined with R in the practical execution of the network as in Fig. 7(c). The values for k and p follow from the star-mesh transformation theorem and are given by:—

$$k = k' = \frac{R_0 S}{2R_0 + S}\,; \quad p = \frac{R_0^2}{2R_0 + S} = \frac{S}{R_0}\,k\,;$$

Resistance k is identical with $\frac{Z_0}{c}$ of the bridged-T network shown in Fig. 1(d) and mentioned in the introduction to this article, while $p + R$ constitute the resistance $Z_0 \frac{c^2-1}{2c}$. The proof of this will be given later.

A bridged-T network with $c > 1$ is therefore obtained by first designing a bridged-T network with $c = 1$, subsequently using the above star-mesh transformation.

It is equally permissible to carry out a transformation from star to mesh, replacing the star formed by R_0, R_0' and R of Fig. 7(a) by a mesh. Combining the horizontal branch of this mesh with S furnishes another derived network with a saving of one resistor. There seems, however, to be no advantage in this method over the transformation from mesh to star.

Perhaps it should be mentioned that, although the derived bridged-T networks have the same iterative parameters as their prototypes, it is not permissible to short-circuit or open-circuit k' in order to obtain an ⊓ or L network in the manner in which Z' was treated in the case of the basic bridged-T network.

2. *Connection between conventions used under* 1 *and notation employing the factor* c.

By definition let $k = \frac{R_0}{c}$; then $c = \frac{R_0}{k}$

but $k = \frac{R_0 S}{2R_0 + S}$; $\therefore c = \frac{2R_0 + S}{S}$;

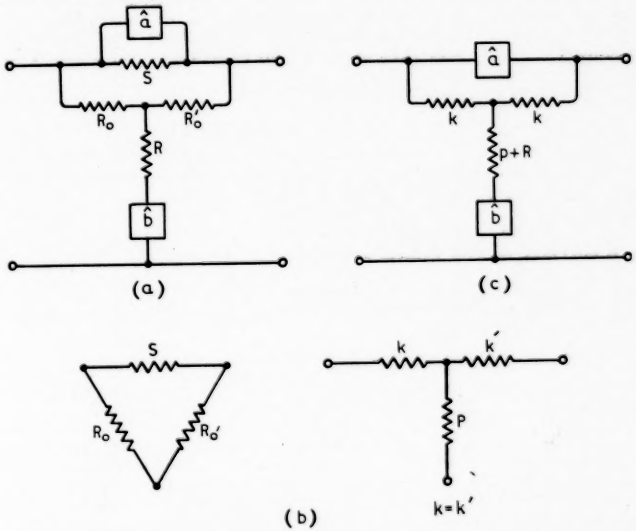

Fig. 7. *In the derived bridged-T network a star-mesh transformation is used to save a component.*

It will be noted that this explains why passive networks with $c < 1$ are physically impossible.

Now :— $p + R = \frac{R_0^2}{2R_0 + S} + R$;

but $RS = R_0^2$ and $R = \frac{R_0^2}{S}$

$$\therefore p + R = \frac{R_0^2}{2R_0 + S} + \frac{R_0^2}{S}$$

$$= R_0 \frac{2R_0 S + 2R_0^2}{2R_0 S + S^2} = R_0 \frac{4R_0^2 + 4R_0 S}{4R_0 S + 2S^2}$$

$$= R_0 \frac{(2R_0 + S)^2 - S^2}{2(2R_0 + S)S} = R_0 \frac{\frac{(2R_0 + S)^2}{S^2} - 1}{2\frac{2R_0 + S}{S}}$$

but $c = \frac{2R_0 + S}{S}$

$$\therefore p + R = R_0 \cdot \frac{c^2 - 1}{2c}$$

This establishes the identities mentioned under (1) of this Appendix.

APPENDIX II

Detailed Design Procedure of Bridged-T Equalizer suitable for a Lump-loaded Line

The design is commenced by concentrating on branch b [Figs. 1(a), (b) and (c)].

For the adjustment of b the circuit of Fig. 8 is set up, consisting of a variable frequency oscillator S, a level measuring set, in "through level" position U* a resistance R_0 equal to the design impedance of the equalizer and variable components L, C, and R forming the branch b of the equalizer under consideration.

The dashes in the subsequent expressions indicate conditions at different frequencies, one dash for the values at the highest frequency of the band considered, two dashes for the lowest frequency.

Let A be the attenuation, at any frequency, of the line for which the equalizer is to be designed.

Let M be the voltage, expressed in db, applied to the experimental circuit, at any frequency, as indicated by the dashes.

Let m be the voltage, expressed in db, measured across the branch b of the network under construction, at any frequency, as indicated by the dashes.

Let n be the desired attenuation of the network at any frequency, as indicated by the dashes.

Normally A will be greatest at the highest frequency considered. The equalizer network will, however, have a "residual attenuation" of n' at this frequency. The equalizer must be so designed that the total attenuation of the line plus equalizer is $A' + n'$ at all relevant frequencies.

First the frequency of the oscillator is set to the highest frequency considered and the tuned circuit adjusted until $M' - m'$ is a minimum. This furnishes $n' = M' - m'$, the value of R being of no practical consequence at this stage.

The oscillator is now set to the lowest frequency considered; the attenuation n'' required at this frequency is

$$(A' + n') - A'' = M'' - m''$$

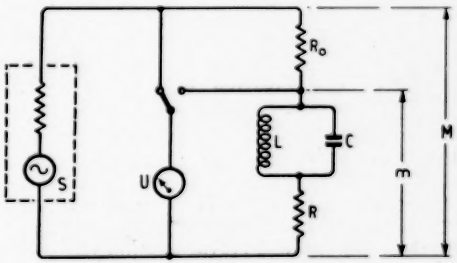

Fig. 8. *Circuit used in determining the constants of an equalizer for a lump-loaded line.*

* A level measuring set is said to be in "through level" position if the conditions set up in the instrument are such that the impedance between its terminals can be considered as infinite.

R is therefore adjusted until $M'' - m'' = (A' + n') - A''$.

The correct attenuation at intermediate frequencies is obtained by altering the ratio $\frac{L}{C}$ without altering $L \times C$ in such a way that $M - m = (A' + n') - A$ within the required tolerance at all relevant frequencies.

As M is adjustable at will the work can be made easier by keeping M to zero db at all frequencies. In this case the above expression becomes :—

$$-m = (A' + n') - A$$

In other words the components of b are then adjusted in such a way that m is numerically equal to the desired attenuation at all frequencies.

Perhaps it is still speedier to make M negative and numerically equal to A at all frequencies, then

$$-A - m = (A' + n') - A$$
$$\therefore m = -(A' + n') = \text{a constant.}$$

This means that all that is necessary in this case is to adjust the tuned circuit at the highest frequency until m' becomes a (negative) minimum, and then carry out all subsequent adjustments in such a way that m remains constant within the tolerances allowed.

CLASS B AUDIO-FREQUENCY AMPLIFIERS*

By F. Butler, B.Sc., A.M.I.E.E.

SUMMARY.—Distortion caused by the variable input impedance of an amplifier in which grid current flows during a part of each cycle of the driving voltage is reduced by the use of an output stage having an input impedance which is very low at all values of signal amplitude. In this case, the additional parallel loading represented by grid-current flow is comparatively small.

An earthed-grid, cathode-coupled amplifier has the desired characteristics and has the further advantage that although it requires considerable excitation power, a large proportion of this appears as useful output. The division of power between the final and penultimate stages is of special importance in the design of high-power modulators, in which it may be difficult to secure the desired output from a single stage.

1. Introduction

At audio frequencies, the input impedance of a conventional triode amplifier is almost infinite, except when the amplitude of the excitation voltage exceeds the standing negative grid bias. The intermittent flow of grid current then constitutes a variable loading on the driver valve, and unless the voltage regulation of this stage is exceptionally good, amplitude distortion becomes serious, due to the clipping of the crests of the positive half-cycles of the driving voltage. Abrupt changes in input impedance also tend to cause undesirable transient oscillations, or "ringing", often difficult to suppress.

Negative feedback can be used to lower the output impedance of the driver valve, at the expense of a corresponding reduction in stage gain. Alternatively a step-down driver transformer may be employed, its turns ratio being subject to the restriction that adequate excitation for the output valves must be available without excessive distortion in the penultimate stage. For the best results, the output power from the driver must be greatly in excess of the minimum excitation power requirements of the final amplifier, the balance being dissipated in a dummy load.

An examination of the properties of the earthed-grid, or cathode-coupled, amplifier shows that it is almost ideally suited to operation under Class B conditions. In the first place, its mean input impedance is very low at all values of signal level, and, in comparison, the maximum variations due to intermittent grid current flow are negligible. Moreover, the high excitation power required is not entirely dissipated as in the conventional system. By proper design, a large fraction of the driver output can be transferred to the final stage. In the case of high-power amplifiers or modulators, the contribution from the driver reduces the requirements from the final stage and thus simplifies design.

2. Cathode-input Amplifier

Fig. 1 shows a triode driver V_1 coupled to an earthed-grid amplifier V_2. Replacing the actual valves by their equivalent generator

*MS. accepted by the Editor, May 1946.

circuits and restricting the analysis to linear conditions, equations may be derived for the overall stage-gain, the final power output and the requisite output from the driver stage. The case of impedance coupling will first be discussed and the necessary modifications for ideal transformer coupling will then be given.

Replacing the actual circuit of Fig. 1 by its electrical equivalent, shown in Fig. 2, the network equations may be derived.

Let E = input voltage to V_1.
E_1 = equivalent generator e.m.f. of V_1, $= \mu_0 E$.
i_1, i_2 = mesh currents in the network.
R_0 = anode slope resistance of V_1.
μ_0 = amplification factor of V_1.
z = effective anode load impedance of V_1, excluding the input impedance of V_2. $= z_1$ and z_2 in parallel.
R_L = load resistance of V_2.
R_a = slope resistance of V_2.
μ = amplification factor of V_2.
R_2 = $R_L + R_a$.
E_c = grid-cathode potential of V_2. = voltage drop across the impedance z.
$E_2 = \mu E_c$.
E_0 = final output voltage.

Applying Kirchhoff's laws, it can be shown that the ratio of the output voltage E_0 to the equivalent generator voltage E_1 of the driver valve is given by :

$$\frac{E_0}{E_1} = \frac{z}{R_0 + z} \cdot \frac{(\mu + 1) R_L}{R_L + R_a + (\mu + 1)\dfrac{R_0 z}{R_0 + z}} \quad (1)$$

The overall stage gain E_0/E is obtained by substituting, in equation (1), $E_1 = \mu_0 E$.

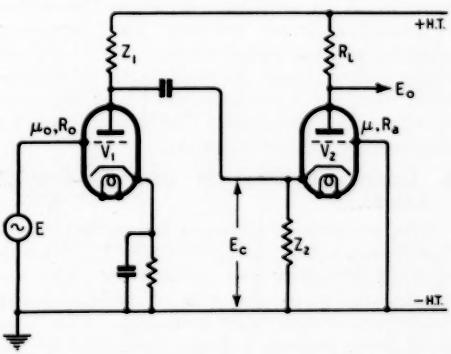

Fig. 1. *Basic circuit of a cathode-input stage and its driver.*

The terms of this equation have been grouped in such a manner as to stress the physical significance of each. First, there is a reduction of gain below the theoretical maximum, due to the finite value of the load impedance z, the extent of the loss being settled by the relative magnitudes of load and valve impedance in the first amplifier stage. The term $(\mu + 1)\dfrac{R_0 z}{R_0 + z}$ represents an increase of effective valve resistance caused by impedance in the cathode circuit of the final stage. The increase is due to a form of negative feedback similar to that which

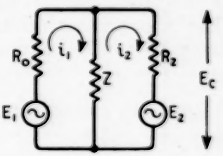

Fig. 2. *Equivalent circuit of the amplifier of Fig. 1.*

is observed in the case of an amplifier with an unshunted cathode-bias resistance. The impedance $\dfrac{R_0 z}{R_0 + z}$ is the parallel combination of the driver-valve resistance and its load impedance, and is the effective output impedance viewed from the cathode of the driven stage. With a fixed value assigned to the load resistance R_L, the maximum gain is obtained when z, the parallel combination of z_1 and z_2, is infinite. For all practical purposes this assumption is permissible if z_1 and z_2 are iron-cored chokes of sufficiently high inductance.

Reference to Fig. 2 shows that, in this special case, the two valves are effectively in series and the currents i_1 and i_2 become identical.

Setting $i_1 = i_2 = i$, with $z = \infty$, the output voltage $E_0 = iR_L$ and equation (1) may be re-written :

$$E_1/i = Z_i = R_0 + \frac{R_a + R_L}{\mu + 1} \quad .. \quad (2)$$

E_1 is the equivalent generator voltage and R_0 the internal output impedance of V_1. The second term of equation (2) is clearly the input impedance of V_2, which forms the load impedance of V_1. The practical design of an amplifier of this kind then reduces to the problem of securing a proper match between the generator impedance R_0 and the load impedance $\dfrac{R_a + R_L}{\mu + 1}$ and to the proper choice of the final load impedance R_L. The latter is selected according to well-established rules and, for standard valves, may be derived from manufacturers' data sheets. There remains the question of matching R_0

to $\frac{R_a + R_L}{\mu + 1}$. In general, direct coupling will be found impracticable and a matching transformer must be used. The insertion of an ideal transformer of turns ratio n between the valves converts the load impedance to the value $Z = n^2 . \frac{R_a + R_L}{\mu + 1}$.

The exact theory of transformer coupling is very complex but subject to the condition that the primary inductance is sufficiently large to make its reactance, at the lowest significant frequency, of the order of ten times the first valve slope resistance, there is little loss of accuracy in considering only the ideal case.

3. Cathode-Follower Driver Valve

It has already been stated that the relative values of the resistances R_0 and $\frac{R_a + R_L}{\mu + 1}$ are in practice such as to preclude the possibility of direct coupling.

It is of interest to consider the use of a cathode-follower as a driver valve. The circuit is shown in a schematic form in Fig. 3.

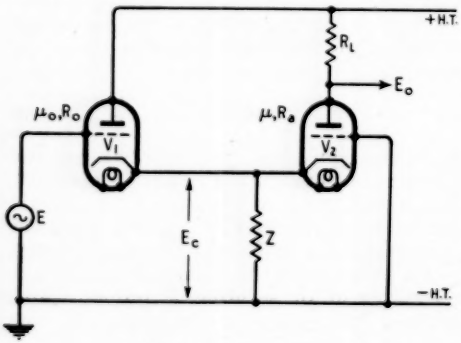

Fig. 3. *Cathode-input amplifier driven by a cathode-follower.*

Normally Z will be an iron-core inductance, and instead of R_L, an output transformer will be used. The modifications for push-pull operation simply involve duplication of the valves and the use of centre-tapped chokes and transformers.

The theory already given is sufficiently general to cover the present case. The cathode-follower valve can be replaced by an electrical equivalent. If μ_0, R_0 are respectively the amplification factor and slope resistance of the valve and if E is the input voltage, the equivalent generator has an e.m.f. $\mu_0/(\mu_0 + 1) . E$ and an internal resistance $R_0/(\mu_0 + 1)$.

Substituting these values in equation (1), the stage gain is given by :—

$$\frac{E_o}{E} = \frac{\frac{\mu_0}{\mu_0 + 1} . Z}{\frac{\frac{R_0}{\mu_0 + 1}}{\frac{\mu_0 + 1}{}} + Z} . \frac{(\mu + 1) R_L}{R_L + R_a + (\mu + 1) \frac{\frac{R_0 Z}{\mu_0 + 1}}{\frac{R_0}{\mu_0 + 1} + Z}} \quad .. \quad .. \quad (3)$$

Assuming μ_0 and $\mu \gg 1$ and setting $Z \to \infty$, equation (3) can be simplified and becomes : $E_0/E = \frac{\mu R_L}{R_L + R_a + \mu/g_m}$, where g_m is the mutual conductance of V_1.

The use of a cathode-follower avoids the necessity for a coupling transformer and a centre-tapped choke may be substituted. The greatest disadvantage is the low gain, which calls for the use of additional pre-amplifier stages. It must also be remembered that the driver and the driven stages are effectively in series and therefore the alternating component of anode current is the same in each. This consideration limits the choice of suitable driver valves. The foregoing remarks are restricted to the use of valves under linear, Class A conditions. Modifications must be introduced to cover the case of Class B operation. These will now be discussed in empirical terms.

4. Design of Class B Cathode—Coupled Amplifiers

The successive steps to be performed in the design of a push-pull Class B cathode-coupled amplifier are as follows. The remarks apply to the use of triodes using interstage transformer coupling. This case is of more general interest than that of the cathode-follower.

(i) Select a pair of valves to give the desired output from the final stage under normal Class B conditions.

(ii) Consult valve tables and determine the anode-to-anode load resistance, the h.t. voltage and negative grid bias.

(iii) Calculate the input impedance of the output valves, $\frac{R_a + R_L}{\mu + 1}$, modifying the result to allow for push-pull Class B operation.

(iv) Determine the r.m.s. excitation voltage required to load the output valves. This information can also be obtained from valve data sheets.

(v) Calculate the maximum-signal driving power required from the penultimate stage to operate the output valves, using the known r.m.s. value of the driving voltage and the known input impedance.

(vi) Choose a pair of driver valves capable of supplying this power with an adequate reserve.

(vii) From valve tables, find the proper load impedance for the driver valves.

(viii) Select the turns ratio and power-handling capacity of the coupling transformer required to convert the input impedance of the output valves to the load value required by the penultimate stage.

In view of the fact that published valve characteristics are the results of static observations, it is preferable to make measurements of the input impedance under working conditions instead of performing the calculation shown in para. (iii) above. This is because the separate valves of a Class B stage are alternately active and quiescent, which complicates the analysis.

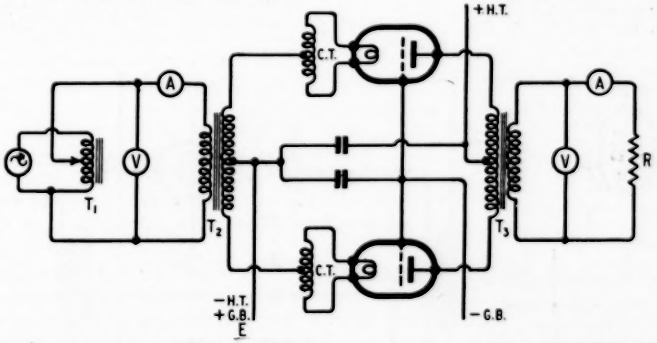

Fig. 4. *Circuit for the measurement of the input impedance of a Class B stage.*

5. Measurement of Input Impedance

A circuit suitable for the measurement of the input impedance of the output valves is shown in Fig. 4, and it is convenient to make measurements at 50 c/s. A "Variac" transformer T_1 is connected to the a.c. mains and its variable output applied to the primary of a coupling transformer T_2, of turns ratio $n : 1$. The secondary centre-tap is earthed and the outer ends of the winding connected respectively to the centre-taps of the separate filament transformer windings which supply the two output valves. Rectifier voltmeters and ammeters permit the measurement of input and output voltages and currents. Exceptionally good regulation of h.t. and grid bias supplies is essential, and impedance measurements should be made over a wide range of input signal level. It will be found that there is a definite drop in the input impedance once the peak driving voltage exceeds the fixed negative bias, but the percentage change remains relatively small, unless the valves are dangerously over-driven.

Let E = measured input voltage to T_2.
I = observed primary current.
Then, input impedance $Z_i = E/I$.

The actual input impedance between the valve cathodes, taking account of the effect of the coupling transformer, is :—

$$Z = n^2 Z_i$$

Once the value of Z is determined, either by measurement or by calculation, the correct ratio of the coupling transformer required to match the driver valves to the input impedance of the final stage is given by :—

$$n = \sqrt{R/Z}$$

where R is the optimum load of the driver stage. At first sight it might seem preferable to make direct input-impedance measurements on the secondary side of the coupling transformer. By using the method described, the transformer losses are included in the measurement and a more realistic figure for the impedance is obtained. If possible, the test transformer should be physically similar to the component which will eventually be employed as a driver.

6. Experimental Results

Using the circuit of Fig. 4, results are obtained which support the theoretical conclusions and indicate that the cathode-

coupled audio-frequency amplifier has useful properties not possessed by normal Class B amplifiers. In making such measurements, the experimental procedure is as follows :—

(i) From valve data sheets select a pair of valves capable of delivering the desired output and choose an appropriate load resistance, allowing for the turns ratio of the output transformer.

(ii) Using a double-beam oscilloscope, display the input voltage to the driver transformer on one trace and the amplifier output voltage on the other.

(iii) Adjust the input and output signal levels to equality and bias the deflector plates until the two displays can be superimposed. Auxiliary potentiometers used for this purpose must be of high resistance in comparison with the load resistance and with the input impedance of the driver transformer. Alternatively, allowance must be made for power dissipated in them.

(iv) Raise the input signal level until the output shows signs of distortion. This is recognised by the fact that the two traces can no longer be brought into coincidence, and is an indication that the maximum useful power output has been attained.

(v) Record the input and output voltages and currents. Note the h.t. supply voltage and feed current.

Results are recorded below for the case of a particular amplifier using 25-watt triodes in the output stage. The valves are apparently being over-run, but although the h.t. voltage is 50 per cent above the makers' rating, the maximum permissible anode dissipation is not exceeded.

Type of valve : Marconi-Osram PX25
H.T. Voltage : 600 V
Feed Current : 170 mA
D.C. Grid Current : 3 mA
Input Power : 102 W
Input Transformer (T_2 in Fig. 4) turns ratio (Primary : Secondary) : 1 : 2.
Output Transformer (T_3 in Fig. 4) turns ratio (Primary : Secondary) : 2.8 : 1.
Load Resistance : 1,100 Ω
Equivalent load resistance at anodes of output valves :
$(2.8)^2 \times 1100 = 8{,}624$ Ω
Output Voltage : 245 V r.m.s.
Output Current : 0.223 A r.m.s.
Output Power (maximum undistorted): 54.63W
Input Voltage : 65 V r.m.s.
Input Current : 0.22 A r.m.s.
Driving Power : 14.3 W
Input Impedance at primary of T_2 : 295.5 Ω
Equivalent Input Impedance (between valve cathodes) :
$2^2 \times 295.5 = 1{,}182$ Ω.

The high ratio of driving power to the final output power is clearly evident. As already pointed out, a proportion of the driving power of 14.3 W is transferred to the output and contributes to the total of 54.63 watts delivered to the load. The apparent conversion efficiency of the amplifier is 54.63/102 = 53.6 per cent. The actual value is lower than this since the excitation power has not been included in the calculations.

If an attempt is made to increase the output power by raising the excitation voltage, there is a rapid increase in grid current, due first to the increased positive grid voltage, and to the simultaneous reduction of mini-

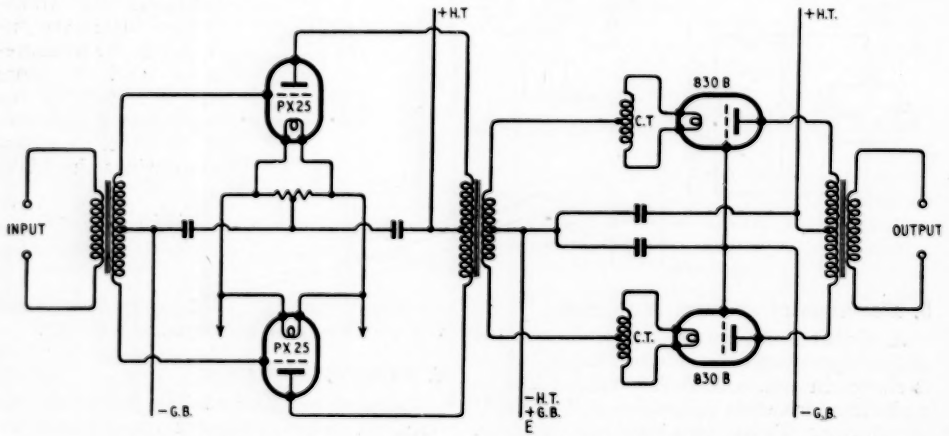

Fig. 5. *Push-pull driver and amplifier stages embodying the principle described.*

mum anode potential due to the larger alternating voltage developed across the load. This is exactly analogous to the case of Class B or Class C radio-frequency amplifiers, in which there is corresponding rapid rise of excitation power once a certain output is exceeded. Correct choice of load resistance minimizes this difficulty at the expense of reduced efficiency.

The above experimental figures do not reveal all the properties of the amplifier. The relationships between driving power and final output, between excitation voltage and grid current, and between input impedance, excitation voltage and load resistance should also be studied. These are straightforward investigations, and will not be described in further detail.

7. Practical Amplifier Circuit

Fig. 5 shows a push-pull driver and output stage employing the principles already described. With the valve types shown, operating at maximum ratings, the output should be nearly 200 watts. Conventional pre-amplifier stages may be added to raise the overall gain to any desired level. If desired, beam-tetrode driver valves may be employed. These have the advantage of higher efficiency, though distortion may be rather greater than with triodes.

To secure a large output with minimum distortion, it is essential that h.t. and grid bias supplies are well regulated. In high-power modulators, supplied from mercury-vapour rectifiers, the voltage regulation is satisfactory. It is usually possible to select gas-discharge regulators for stabilizing the grid-bias supplies of medium power amplifiers. Alternatively, electronic regulators must be used. It is also of importance to keep the driver-transformer secondary windings of low resistance, otherwise an undesired variable cathode bias will be introduced, due to the fluctuating anode current of the output valves.

No reference has been made to the use of negative feedback in connection with cathode-driven amplifiers. Any of the usual systems may be employed, though voltage feedback will normally be preferred, in order to reduce the output impedance of the amplifier. Reference to equation (1) shows that the normal valve resistance R_a is increased to a value :—

$$R_a + (\mu + 1)\frac{R_0 z}{R_0 + z}.$$

In transmitter modulators, this increase is not objectionable, but in public address amplifiers the loudspeaker damping is not quite so good as with the usual Class B system. Voltage feedback can correct this defect and simultaneously reduce the residual distortion.

As regards the response at high audio frequencies, the cathode-driven valve has an inherent advantage over other triode amplifiers, since the earthed grid avoids degenerative feedback due to anode-grid capacitance. To obtain a comparable performance with normal amplifiers, audio-frequency neutralizing would be necessary.

Professor G. W. O. Howe

On his retirement from the James Watt Chair of Electrical Engineering in the University of Glasgow, Professor Howe was presented with a cheque for £200, the presentation being carried out by Professor Bernard Hague.

Professor Howe has handed the gift to the Principal of the University for the endowment of a prize for the most outstanding graduate in electrical engineering.

R. N. Vyvyan

We regret to announce the death of R. N. Vyvyan on December 14th, at the age of 70. Formerly Engineer-in-Chief of the Marconi Company, Mr. Vyvyan was one of the pioneers of wireless. He supervised the erection of the famous Poldhu and Cape Cod stations.

While Managing-Engineer in Canada for the Marconi Company he built the Glace Bay station, which formed the western link of the first regular transatlantic wireless service. He returned to this country in 1908.

He was responsible for the construction and engineering design of the short-wave beam system of the Imperial wireless stations which came into being after the first World War. He retired in 1936.

Popov Medal

Instituted by a decree of the Soviet Government, the Popov Gold Medal is awarded annually on 7th May by the Academy of Sciences of the U.S.S.R. to a scientist of any nationality for outstanding scientific research or invention in the field of radio. The first award will be made in 1947 for work completed in the period 1933–1945 and work submitted must reach the Academy of Sciences Radio Physics and Radio Engineering Council not later than 1st February.*

Papers may be in any language, typed or printed, and there should be three copies. The address of the Council is Tretya Miusskaya d 3, Moscow, U.S.S.R

*Notification of this competition was not received until early December, and the early closing date would seem to make entries from this country difficult.—Ed.

TRANSIENT RESPONSE OF V.F. COUPLINGS*

Compensated Networks for Wide-Band Amplifiers

By W. E. Thomson, M.A.

(*P.O. Radio Branch*)

SUMMARY.—Formulae and curves are given for the response to the Heaviside unit function of a single resistance-capacitance coupled stage as commonly used in wide-band amplifiers, the nominally resistive load being compensated to offset the low- and high-frequency deficiencies associated with this type of coupling.

The analysis of the low-frequency response deals mainly with the compensation of grid coupling by anode decoupling. Two orders of compensation are dealt with.

For high-frequency compensation, one particular type of compensating circuit is analysed. Attention is confined to conditions which give "critical damping." The ultimate possible improvement with this type of circuit is demonstrated.

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   2.1 General
   2.2 Compensation of grid coupling by anode decoupling
3. High-Frequency Compensation
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   3.2 Particular cases
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   2.1 General
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References

1. Introduction

FOR amplifiers, such as oscillograph and video amplifiers, which must pass a signal with as little change of waveform as possible, the amplitude-frequency and phase-frequency responses do not give an entirely satisfactory criterion of performance. It is preferable, and now fairly general, to take as a criterion of performance the indicial response ; that is, the response to a Heaviside unit function.

In this article, several commonly used types of coupling are analysed. The work is of a general nature and includes previous results as particular cases. Attention has been confined to the theoretical aspects, and the numerous practical considerations involved are not discussed.

The usual assumptions have been made. These are :—

(1) The valve concerned can be regarded as a constant-current generator.

(2) The high-frequency response can be considered separately from the low-frequency response. For each case an equivalent circuit is used which neglects certain impedances. As far as the indicial response is concerned, this distinction corresponds to a distinction between the initial build-up period, which is associated with the high-frequency response, and the maintenance of the response at the ideal level up to a certain maximum period, which is associated with the low-frequency response.

(3) The impedance of the d.c. supply is negligible.

The symbols used are either standard or are defined when required ; certain symbols, such as α, β, have different meanings in different sections.

2. Low-Frequency Compensation

2.1. *General*

For an amplifier which cannot amplify d.c. the indicial response must tend to zero with increasing time. It is possible, however, to design the amplifier so that the response may approximate the ideal, within given limits, up to a certain maximum time. Let the indicial response be expressed in a Maclaurin series in t :

$$V = 1 + a_1 t + a_2 t^2/2\,! + a_3 t^3/3\,! + \ldots$$

* MS. accepted by the Editor, April 1946.

Then the response may be made more and more nearly ideal by choosing the design parameters so that

$$a_1 = a_2 = a_3 = \ldots = a_n = 0$$

for n as large as possible.

The corresponding expression for the operational gain is[1]

$$G = 1 + a_1/p + a_2/p^2 + a_3/p^3 + \ldots$$

The above condition may also be expressed in terms of derivatives, thus :

$$\left[\frac{dV}{dt}\right]_0 = \left[\frac{d^2V}{dt^2}\right]_0 = \ldots = \left[\frac{d^nV}{dt^n}\right]_0 = 0,$$

or

$$\left[\frac{dG}{d\,(1/p)}\right]_0 = \left[\frac{d^2G}{d\,(1/p)^2}\right]_0 = \ldots = \left[\frac{d^nG}{d\,(1/p)^n}\right]_0 = 0$$

In practical work it is usually sufficient to make $a_1 = 0$ only, and the conditions for this can be determined fairly readily from the operational gain. For example the normalized gain for the circuit of Fig. 1 is*

$$G = \frac{1 + Z_d/R_L}{(1 + g_cZ_c + \sigma_sZ_s)\,(1 + 1/pC_gR_g)}$$

where g_c = sum of grid-anode, grid-screen, and screen-anode mutual conductances,

σ_s = screen conductance,

Z_d, Z_c, Z_s are anode decoupling, cathode, and screen networks,

and it is assumed that the coupling capacitor and grid leak can be regarded as of high impedance.

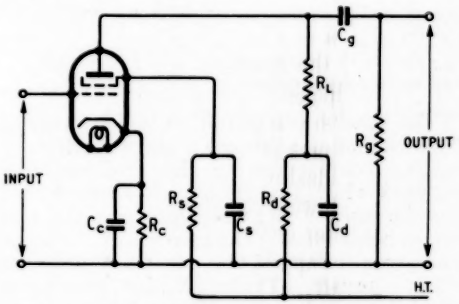


Fig. 1. *Typical resistance-capacitance coupled stage including decoupling, screen-feed, and cathode-bias networks.*

Each of the networks Z_d, Z_c, and Z_s consists effectively of a resistance in parallel with a capacitance. The operational impedance is of the form :

$$\frac{R/pC}{R + 1/pC} = \frac{1/pC}{1 + 1/pCR}$$
$$= (1/pC)\,[1 - 1/pCR + 1/(pCR)^2 \ldots]$$
$$= 1/pC \text{ to a first approximation.}$$

Hence, to a first approximation :

$$G = \frac{1 + 1/pC_dR_L}{(1 + g_c/pC_c + \sigma_s/pC_s)(1 + 1/pC_gR_g)}$$
$$= (1 + 1/p)\,(1/C_dR_L - g_c/C_c - \sigma_s/C_s - 1/C_gR_g)$$

Thus the condition which makes $\left[\frac{dV}{dt}\right]_0 = 0$ is

$$1/C_dR_L = g_c/C_c + \sigma_s/C_s + 1/C_gR_g$$

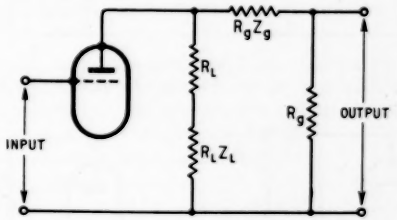


Fig. 2. *The basic coupling circuit ; R_LZ_L and R_gZ_g are networks which respectively do and do not pass direct current.*

2.2.—*Compensation of grid coupling by anode decoupling.*

A case frequently considered is illustrated in Fig. 2 in which R_L and R_g are the anode load and grid leak respectively ; R_LZ_L is a decoupling network which must pass d.c. and R_gZ_g is a coupling network which does not pass d.c. The impedance of both networks tends to zero with increasing frequency. The normalized operational gain of such a stage, assuming that the grid network can be regarded as a high-impedance potential divider across the effective load is :

$$G = \frac{1 + Z_L}{1 + Z_g}$$

Obviously if $Z_L = Z_g$ the gain will be independent of frequency and the indicial response perfect. The d.c. requirements, of course, make this condition impossible. It is possible, however, by making Z_L and Z_g almost equal, to satisfy the conditions stated in the previous section. Z_L and Z_g have the form shown in Fig. 3. These forms automatically fulfil the d.c. and high-frequency conditions stated above. In Fig. 3, Z_L and

* Adapted from a formula of Edwards and Cherry.[9]

Z_g have n components identical; in 3(a), Z_g has an extra component C_{n+2} and in 3(b), Z_L has the extra component R_{n+2}. In both cases, as is shown in Appendix 2.1,

$$\left[\frac{dV}{dt}\right]_0 = \left[\frac{d^2V}{dt^2}\right]_0 = \ldots = \left[\frac{d^nV}{dt^n}\right]_0 = 0$$

$$n = 0; \qquad V = e^{-t/T}$$

$$n = 1; \qquad V = \frac{1}{\alpha - 1}\left(e^{-t/\alpha T} - e^{-t/T}\right)$$

where $T = C_dR_L = C_gR_g$, and α, as is shown in Fig. 4, is the ratio of decoupling to load resistance.

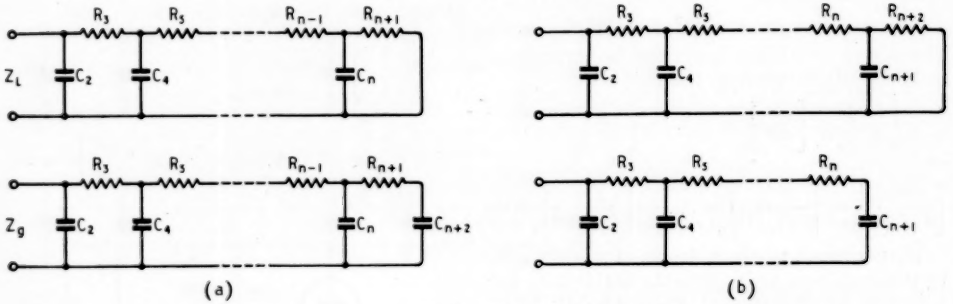


Fig. 3. *General forms of Z_L and Z_g of Fig. 2 are shown here.*

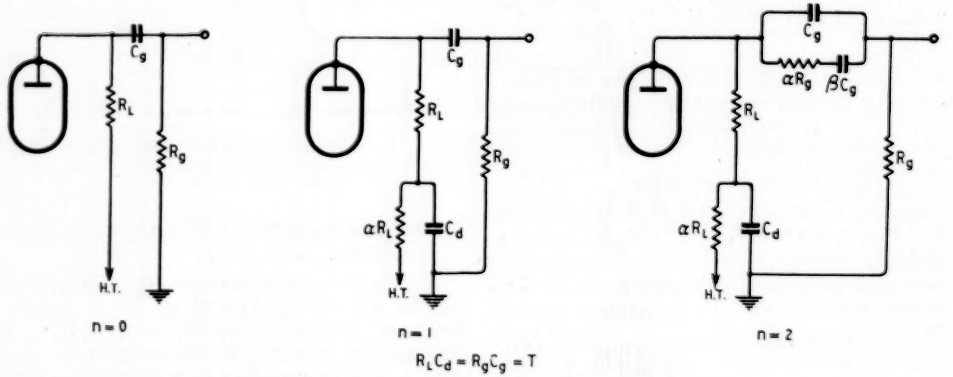


Fig. 4. *The three simplest practical forms of Fig. 2: $n = 0$, uncompensated; $n = 1$, first-order compensation; $n = 2$, second-order compensation.*

Fig. 4 shows the three simplest cases, for $n = 0$, 1, and 2 respectively; the components have been renamed for convenience. Case $n = 0$ is the basic resistance-capacitance coupled stage. For case $n = 1$, the anode load has been modified. The addition of the capacitor C_d alone, with value C_gR_g/R_L would give perfect response; the capacitor must however be bypassed by αR_L to allow h.t. current to flow. The correct relationship $C_dR_L = C_gR_g$ gives an indicial response for which $[dV/dt]_0 = 0$ as is shown in Fig. 5, which gives responses for various values of α; the curve $\alpha = 0$ corresponds to the case $n = 0$. This circuit has been adequately analysed elsewhere [10], [11], so only the formulae are given here;

The case $n = 2$ is not so widely known. This carries the process a stage further by adding components to the grid coupling network which compensates, to some extent, for the resistor αR_L which prevents the response being ideal in the case $n = 1$. Perfect compensation would be achieved by shunting C_g by a resistor of value αR_g, but, in order to block h.t., a capacitor βC_g must be added in series with αR_g. The addition of these two components does not affect $[dV/dt]_0$: provided $C_dR_L = C_gR_g$ we still have $[dV/dt]_0 = 0$. The correct value of resistor, namely αR_g, improves the indicial response still further, by making $[d^2V/dt^2]_0 = 0$. This is shown in Fig. 6 which gives responses for various values of α and β. The curves $\beta = 0$ corres-

pond to the case $n = 1$. The curves also show that it is better to increase α, rather than β. The formula for this case is derived in Appendix 2.2 and given below:

$n = 2$;

$$V = \frac{\alpha\beta}{\alpha - \alpha\beta - 1}\left[\frac{1}{\beta}e^{-t/\alpha T} + \frac{q(1+mq-m)}{m-q}e^{-mt/T} - \frac{m(1+mq-q)}{m-q}e^{-qt/T}\right],$$

where $m = c + d \quad c = \frac{1}{2}(1 + 1/\alpha + 1/\alpha\beta)$

$q = c - d \quad d^2 = c^2 - 1/\alpha\beta$

$T = C_d R_L = C_g R_g$

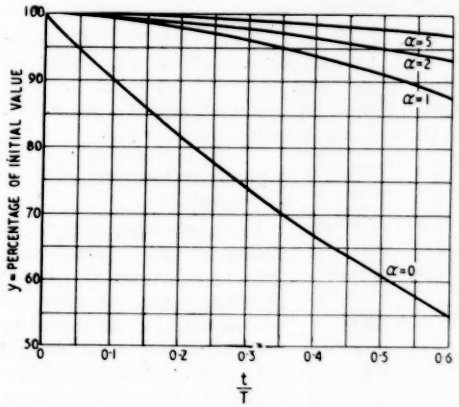


Fig. 5. *Indicial response for first-order l.f. compensation.*

3. High-Frequency Compensation

3.1.—*General*

Of the many types of compensated networks only one is considered here; it is shown in Fig. 7. The nominal load is R_L, and C_s is the total shunt capacitance across the effective load. The compensation is effected by a two-terminal network $R_L Z$ in series with R_L. This network is assumed to be purely reactive for the purposes of design; it may have as many components as desired, each extra component

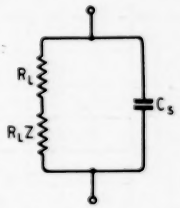


Fig. 7. *Basic network for high-frequency compensation.*

enabling some improvement in response to be obtained. The form of Z must be such that its impedance, as a function of frequency, has a zero at the origin, so that the low-frequency response is unaffected.

The change of shape of the indicial response due to the addition of such compensating networks consists, in general, of reduced build-up time, and oscillation about the steady-state value, this oscillation being generally referred to as "overshoot"* For a given network, improvement in build-up time is obtained at the expense of in-

* In certain types of response, overshoot occurs without oscillation.

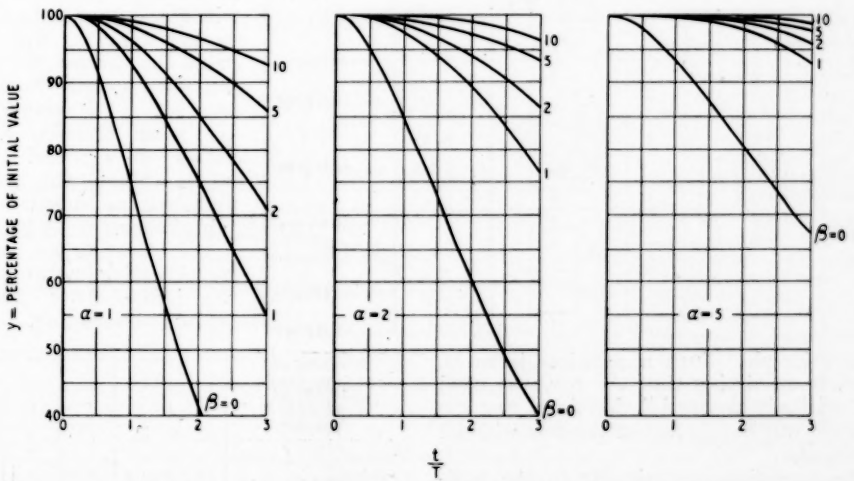


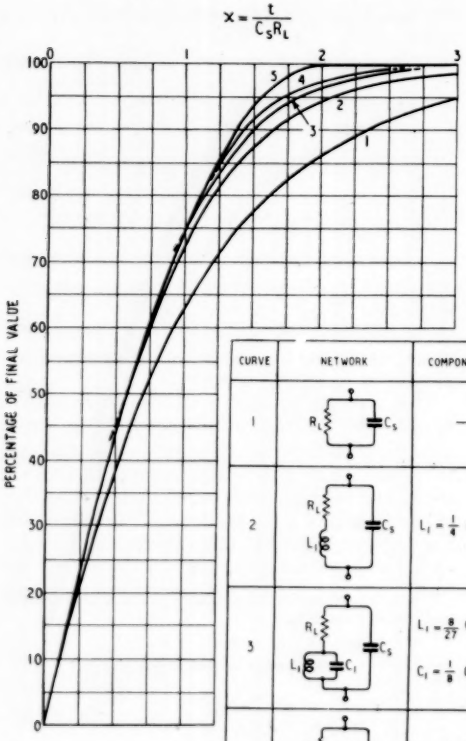
Fig. 6. *Indicial response for second-order l.f. compensation.*

creasing the overshoot once the critical value has been reached.

When Z consists of only a few components, it is possible to make an analysis of all the possible values to determine suitable values for the parameters[4,5,6,7,8,12].

A general treatment requires some simplifying principle. Such a principle is to be found in specifying that the response shall be "critically damped," that is to say the response will be the fastest possible without overshoot.

The conditions are similar to the well-known case of critical damping in a series-tuned circuit where the critical condition is associated with equal real roots of a quadratic equation. Here the nature of the response is determined by the roots of a polynomial equation which is of degree one higher than the number of components in the compensating network. A pair of complex roots gives an oscillatory term in the response, while a real root gives an aperiodic term. Critical damping requires that all the roots be real and equal, and the imposition of this condition is sufficient to determine all the components of the compensating network.



CURVE	NETWORK	COMPONENTS	EQUATIONS & VALUES
1	R_L, C_s	—	x = 0·5 1 1·5 2 $1-e^{-x}$ 39·4 63·2 77·7 86·5
2	R_L, L_1, C_s	$L_1 = \frac{1}{4} C_s R_L^2$	$1-(1+x)e^{-2x}$ 44·7 72·9 87·6 94·5
3	R_L, L_1, C_1, C_s	$L_1 = \frac{8}{27} C_s R_L^2$ $C_1 = \frac{1}{8} C_s$	$1-(1+2x+2x^2)e^{-3x}$ 44·2 75·1 90·6 96·8
4	R_L, L_2, L_1, C_1, C_s	$L_1 = \frac{1}{4} C_s R_L^2$ $C_1 = \frac{1}{4} C_s$ $L_2 = \frac{1}{18} C_s R_L^2$	$1-(1+3x+4x^2+\frac{16}{3}x^3)e^{-4x}$ 43·6 75·6 91·9 97·8
5	R_L, INFINITE, C_s	—	$x-\frac{1}{4}x^2$ 43·8 75·0 93·8 100·0

Fig. 8. *Indicial response of the coupling networks shown for the condition of critical damping.*

3.2.—*Particular cases*

Fig. 8 gives response curves and component values for networks consisting of one, two, and three components. The derivation of the two-component case, which is typical, is given in Appendix 3.1.

3.3.—*Maximum possible improvement*

In order to determine whether any worth-while improvement can be got by increasing the number of components, it is useful to determine the response for a network with an infinite number of components. This has been done in Appendix 3.2 and the curve is included in Fig. 8. It will be seen that up to about 75 per cent of the final value the various curves do not differ greatly from one another. The differences are most pronounced for values of t round about $2C_sR_L$ where the values obtained are shown in the table below.

The form taken by the compensating network for the infinite-order case is rather interesting. The

Number of components in compensating network	0	1	2	3	∞
Percentage of final value	86.5	94.5	96.8	97.8	100

network is equivalent to a negative capacitance of value C_s in series with an open-ended transmission line, impedance R_L, delay C_sR_L. The line is thus matched at the input so that there is only one reflection. The combination of input and reflected waveforms give ideal response after a time $2C_sR_L$.

Fig. 9 shows one form of the infinite order compensating network.

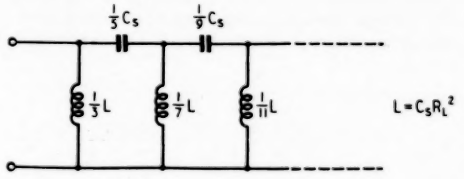


Fig. 9. *One form of the infinite-order compensating network.*

3.4.—*Attenuation and delay-frequency response.*

Figs. 10 and 11 give respectively the attenuation - frequency and delay-frequency characteristics for the five cases dealt with in Fig. 8. The formulae are given in Appendix 3.3.

Acknowledgments

The author is indebted to the Engineer-in-Chief, G.P.O., for permission to publish this article, and to those of his colleagues who have helped in its preparation.

APPENDICES

1. Operational Formulae [1]

General Formulae.

If $\phi(p) \; \mathbf{1} = f(t)$

then $\phi(p/a) \; \mathbf{1} = f(at)$ for a real > 0;

$e^{-ap}\,\phi(p) \; \mathbf{1} = f(t-a)$ for $t > a$; $a > 0$.

Standard Forms.

$$\frac{p}{(p-a)^n}\,\mathbf{1} = \frac{t^{n-1}e^{at}}{(n-1)!} \qquad n > 0$$

$$p^{-n}\,\mathbf{1} = t^n/n! \qquad n > -1$$

2. Compensation of grid coupling by anode decoupling

2.1. *General.*

The network of Fig. 3 (a) will be treated in detail; that of Fig. 3 (b) is essentially similar.

We are concerned with two impedances $1 + Z_L$ and $1 + Z_g$ each consisting of a unit resistance in series with Z_L or Z_g respectively. These

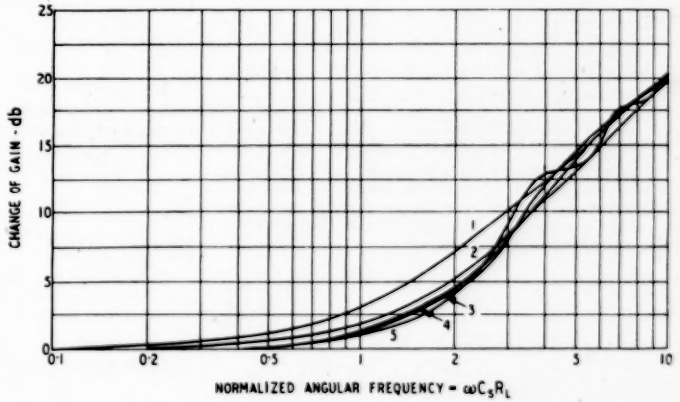


Fig. 10 (*above*). *Amplitude-frequency response curves of the networks of Fig. 8.*

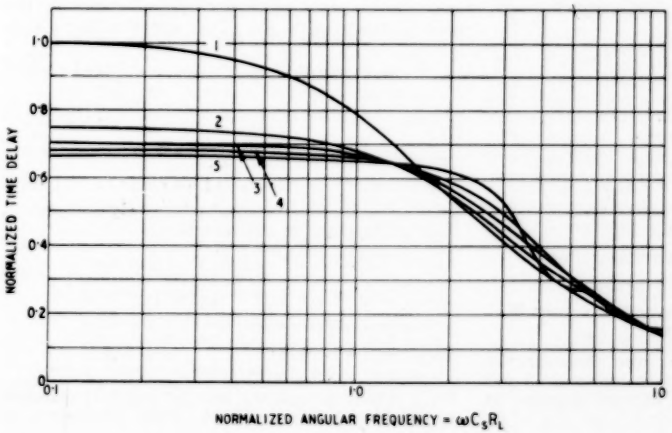


Fig. 11 (*left*). *Delay-frequency characteristics of the circuits of Fig. 8. In each case—Curve 1, uncompensated; curve 2, first-order compensation; curve 3, second-order compensation; curve 4, third-order compensation; curve 5, infinite-order compensation.*

may be expressed as the ratio of two polynomials in p^2.

$$1 + Z_L = \frac{a_0 + a_1 p + \ldots\ldots + a_{\frac{1}{2}n} p^{\frac{1}{2}n}}{1 + b_1 p + \ldots\ldots + b_{\frac{1}{2}n} p^{\frac{1}{2}n}}$$

$$1 + Z_g = \frac{1 + c_1 p + \ldots\ldots + c_{\frac{1}{2}n+1} p^{\frac{1}{2}n+1}}{d_1 p + \ldots\ldots + d_{\frac{1}{2}n+1} p^{\frac{1}{2}n+1}}$$

$$\therefore \quad \frac{1 + Z_L}{1 + Z_g} = \frac{a_0 d_1 p + \ldots\ldots\ldots + a_{\frac{1}{2}n} d_{\frac{1}{2}n+1} p^{n+1}}{1 + (b_1 + c_1) p + \ldots + b_{\frac{1}{2}n} c_{\frac{1}{2}n+1} p^{n+1}}$$

Apart from the unity term in the denominator, numerator and denominator are equal. This can be shown by general continued fraction theory.[3] $1 + Z_L$ and $1 + Z_g$ can be considered as the $(n + 1)$th and $(n + 2)$th convergents of the same continued fraction :

$$1 + \cfrac{1}{C_2 p + \cfrac{1}{R_3 + \cfrac{1}{C_4 p + \ldots \cfrac{1}{R_{n+1} + \cfrac{1}{C_{n+2} p + \ldots}}}}}$$

$$\therefore 1 + Z_L = p_{n+1}/q_{n+1}$$

where p_r and q_r are as defined in reference (3) and are, in this case, polynomials in p ; similarly

$$1 + Z_g = p_{n+2}/q_{n+2}$$

$$\therefore \quad \frac{1 + Z_L}{1 + Z_g} = \frac{p_{n+1} q_{n+2}}{p_{n+2} q_{n+1}}$$

Now, in general,

$$p_n q_{n-1} - p_{n-1} q_n = (-1)^n$$

$$\therefore \quad p_{n+2} q_{n+1} - p_{n+1} q_{n+2} = (-1)^{n+2} = 1, \text{ since, in this case, } n \text{ is even.}$$

$$\therefore \quad \frac{1 + Z_L}{1 + Z_g} = \frac{p_{n+1} q_{n+2}}{1 + p_{n+1} q_{n+2}}$$

Comparing this with the previous expression we see that $p_{n+1} q_{n+2}$ must be identified with the numerator and that the denominator must be equal the numerator plus one. Now convert to a ratio of polynomials in $1/p$:

$$\frac{1 + Z_L}{1 + Z_g} = \frac{F}{F + 1/p^{n+1}} \quad \text{where } F \text{ is a polynomial in } 1/p$$

$$= \frac{1}{1 + 1/Fp^{n+1}}$$

$$= 1 - 1/Fp^{n+1} + 1/(Fp^{n+1})^2 - \ldots$$

The lowest term in $1/p$ in this expression is of degree $n + 1$ therefore, for the corresponding indicial response :

$$\left[\frac{dV}{dt}\right]_0 = \left[\frac{d^2V}{dt^2}\right]_0 = \ldots\ldots = \left[\frac{d^nV}{dt^n}\right]_0 = 0$$

2.2. Special case ; $n = 2$

normalized operational gain

$$= \frac{1 + Z_L}{1 + Z_g} = \frac{pT(1 + \alpha + \alpha pT)(1 + \beta + \alpha\beta pT)}{(1 + \alpha pT)[1 + (1 + \beta + \alpha\beta)pT + \alpha\beta p^2T^2]}$$

after simplification

where α, β, and T are as defined in Fig. 4.

The denominator may be factorized thus :

$$\alpha\beta(p + 1/\alpha T)(p + m/T)(p + q/T)$$

where $m = c + d$ $\qquad c = \frac{1}{2}(1 + 1/\alpha + 1/\alpha\beta)$

$q = c - d$ $\qquad d^2 = c^2 - 1/\alpha\beta$

the gain may therefore be split into three partial fractions each of the standard form $kp/(p + a)$ and hence interpreted to give the indicial response as quoted in section 2.2.

3. High-Frequency Compensation—Critically-Damped Response

3.1. Second order.

Normalized operational impedance of load

$$= \frac{\left(R_L + \dfrac{pL_1}{1 + p^2L_1C_1}\right) \cdot \dfrac{1}{pC_s}}{R_L\left(R_L + \dfrac{1}{pC_s} + \dfrac{pL_1}{1 + p^2L_1C_1}\right)}$$

$$= \frac{1 + \alpha x + \alpha\beta x^2}{1 + x + \alpha(1 + \beta)x^2 + \alpha\beta x^3}$$

where $pC_sR_L = x$, $L_1 = \alpha C_sR_L^2$, $C_1 = \beta C_s$

For critical damping, denominator $= (1 + bx)^3$

hence, comparing coefficients, we have $b = \frac{1}{3}$, $\alpha = \frac{8}{27}$, $\beta = \frac{1}{8}$

So impedance $= \dfrac{1 + \frac{8}{27}x + \frac{1}{27}x^2}{(1 + x/3)^3}$

$$= \frac{x^2 + 8x + 27}{(x + 3)^3} = 1 - \frac{x(x^2 + 8x + 19)}{(x + 3)^3}$$

$$= 1 - \frac{x}{x + 3} - \frac{2x}{(x + 3)^2} - \frac{4x}{(x + 3)^3}$$

Each of these terms may be interpreted by standard forms to give the indicial response

$$V = 1 - (1 + 2t/T + 2t^2/T^2)\, e^{-3t/T}$$

where $T = C_sR_L$

3.2. Infinite order.

For n components the operational impedance Z of the compensating network will be of the form

$$Z = \frac{a_1x + a_3x^3 + \ldots\ldots\ldots + a_{n-1}x^{n-1}}{1 + a_2x + \ldots\ldots\ldots\ldots + a_nx^n}$$

where $x = pC_sR_L$ and the a's are coefficients specifying Z ; n has arbitrarily been taken as even. For this value of Z the normalized impedance of the load becomes

$$\frac{1 + a_1x + a_2x^3 + \ldots\ldots\ldots\ldots + a_nx^n}{1 + x + (a_1 + a_2)x^2 + a_2x^3 + (a_3 + a_4)x^4 + \ldots\ldots\ldots\ldots + (a_{n-1} + a_n)x^n + a_nx^{n+1}}$$

For critical damping, denominator $= \left(1 + \dfrac{x}{n + 1}\right)^{n+1}$

As $n \to \infty$ the impedance becomes the ratio of two infinite series, and for critical damping the denominator equals e^x. This specifies the a's as follows :

$$a_1 + a_2 = 1/2\,! \qquad \text{whence} \quad a_1 = 1/2\,! - 1/3\,!$$

$$a_2 = 1/3\,!$$

$$a_3 + a_4 = 1/4\,! \qquad a_3 = 1/4\,! - 1/5\,!$$

etc.

The numerator therefore is :

$$1 + x/2\,! - x/3\,! + x^2/3\,! + x^3/4\,! - x^3/5\,! + \ldots$$

This series can be shown to be absolutely convergent. It may therefore be rearranged :

$$1 + (x^2/2\,! + x^3/3\,! + \ldots)/x - (x^3/3\,! + x^5/5\,! + \ldots)/x^2$$

$$= 1 + (e^x - 1 - x)/x - (\tfrac{1}{2}e^x - \tfrac{1}{2}e^{-x} - x)/x^2$$

$$= e^x/x - e^x/2x^2 + e^{-x}/2x^2$$

So the impedance is

$$1/x - 1/2x^2 + e^{-2x}/2x^2$$

Each of these terms may be interpreted by standard forms. The factor e^{-2x} in the third term delays its

contribution to the indicial response, which must be considered in two time intervals :

(1) $0 \leqslant t/T \leqslant 2 \qquad T = C_s R_L$
$V = t/T - t^2/4T^2$

(2) $2 \leqslant t/T \leqslant \infty$
$V = t/T - t^2/4T^2 + (t/T - 2)^2/4 = 1$

The curve is shown in Fig. 8

Z may be obtained from the equation

$$\text{impedance} = \frac{1 + Z}{1 + x(1 + Z)}$$

We find that $Z = \coth x - 1/x$;

coth x is the impedance of an open-ended transmission line and $-1/x$ that of a negative capacitance. The network of Fig. 9 is readily obtained from the continued fraction expansion of coth x

$$\coth x = 1/x + \frac{1}{3/x + \frac{1}{5/x + \ldots}}$$

3.3. *Attenuation and Delay Characteristics.*

Order of Compensation	Attenuation Characteristics	Delay Characteristics
0	$10 \log_{10} (1+x^2)$	$(\tan^{-1}x)/x$
1	$20 \log_{10} (4+x^2) - 10 \log_{10} (16+x^2)$	$\{2 \tan^{-1} \frac{1}{2}x - \tan^{-1} \frac{1}{4}x\}/x$
2	$30 \log_{10} (9+x^2) - 10 \log_{10} \{(27-x^2)^2+64x^2\}$	$\{3 \tan^{-1} \frac{1}{3}x - \tan^{-1} \frac{8x}{27-x^2}\}/x$
3	$40 \log_{10} (16+x^2) - 10 \log_{10} \{(256-16x^2)^2 + (80x-x^3)^2\}$	$\{4\tan^{-1} \frac{1}{4}x - \tan^{-1} \frac{80x-x^3}{256-16x^2}\}/x$
∞	$20 \log_{10} x^2 - 10 \log_{10} \{(1 - \cos 2x)^2 + (2x - \sin 2x)^2\}$	$(\tan^{-1} \frac{2x - \sin 2x}{1 - \cos 2x})/x$

$x = \omega C_s R_L$

The delay formulae are for the normalized delay; i.e., ratio of delay to time constant $C_s R_L$.

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CORRESPONDENCE

Transient Response of Filters

To the Editor, "Wireless Engineer"

SIR,—In the March issue D. G. Tucker[1] attempts to find mathematically the transient response of a "Six-element symmetrical" filter section. The ordinary methods of solution prove to give only a guide as to the nature of the solution, and an indirect method of finding the constants involved in the final solution has to be resorted to. Further a number of approximations have to be made.

C. C. Eaglesfield[2] in an article in the November issue attempts and obtains a solution which relies on the bandwidth being small compared with the mid-band frequency.

Acting on a remark made by Tucker in a footnote (page 87) an attempt was made to line-up the transient response of a band-pass network with that of its low-pass analogue.

A low-pass network, which includes components $R_1, R_2, \ldots, L_1, L_2 \ldots, C_1, C_2 \ldots$, has a transfer impedance $Z(\omega)_{LP}$ which may be written

$$Z(\omega)_{LP} = f\left\{R_1,\ R_2,\ \ldots,\ j\omega L_1,\ j\omega L_2,\ \ldots,\ \frac{1}{j\omega C_1},\ \frac{1}{j\omega C_2},\ \ldots\right\}$$

Now if in series with every inductance L we place a capacitance $\frac{1}{\omega_0^2 L}$, and in parallel with every capacitance C we place an inductance $1/\omega_0^2 C$ we obtain the band-pass analogue. This is just a statement of Landon's Low-Pass—Band-Pass Analogue Theorem. For

$$j\omega L \text{ becomes } j\omega L + \frac{1}{j\omega\left(\frac{1}{\omega_0^2 L}\right)} = j\omega L\left(1 - \frac{\omega_0^2}{\omega^2}\right)$$

and $\frac{1}{j\omega C}$ becomes $1 \Big/ \left\{ j\omega C + \frac{1}{j\omega\left(\frac{1}{\omega_0^2 C}\right)} \right\}$

$$= 1 \Big/ \left\{ j\omega C \left(1 - \frac{\omega_0^2}{\omega_0^2}\right) \right\}$$

Thus if we replace ω for the low-pass case by $\omega\left(\frac{1 - \omega_0^2}{\omega^2}\right)$ we have the band-pass case. However, this just states the analogy for the amplitude and phase frequency characteristics. We may demonstrate the analogy for a transient.

By Fourier's Integral Theorem the frequency characteristic of unit step is

$$g(\omega) = \frac{1}{2\pi}\int_0^\infty e^{-j\omega t} \cdot dt = \frac{1}{2\pi j\omega}$$

while the frequency response for the wave $\cos \omega_0 t$. **1** is

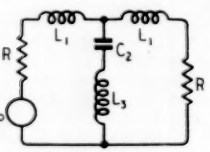
$$g(\omega) = \frac{1}{2\pi}\int_0^\infty \cos \omega_0 t \cdot e^{-j\omega t} \cdot dt = \frac{1}{2\pi j} \frac{\omega}{\omega^2 - \omega_0^2}$$

Comparing these two cases, we see again that the analogy is obtained by replacing ω by $\omega\left(1 - \frac{\omega_0^2}{\omega^2}\right)$.

Thus we may state that if the wave $\cos \omega_0 t$. **1** is put into a band-pass network then the envelope of the output wave is exactly the same as the output

wave obtained when unit step is placed into the low-pass analogue. This is a result (with no restrictions on bandwidth) that should prove of utmost use in the solving of transient problems.

For the "Six-element symmetrical" filter the equivalent low-pass network is clearly (using D. G. Tucker's notation)



where $n\omega_0 = \omega_1$

$$L_1 = \frac{mR}{\omega_1}$$

$$C_2 = \frac{2m}{R\omega_1}$$

$$L_3 = \frac{(1 - m^2)R}{2m\omega_1}$$

When E_0 is unit step, the voltage appearing across R is easily found to be given by

$$V_0 = \tfrac{1}{2}\left\{1 - e^{-\frac{\omega_1 t}{2m}}\left(\cos\frac{\omega_1 t}{2} - \frac{m}{\sqrt{1-\frac{m^2}{4}}}\cdot\sin\left(\frac{\omega_1}{2}\cdot\sqrt{1-\frac{m^2}{4}}\cdot t\right)\right)e^{-\frac{m\omega_1 t}{4}}\right\}$$

which corresponds to equation (7) in C. C. Eaglesfield's article.

London, E.14. E. T. EMMS.

FILTERS

[1] D. G. Tucker. "Transient Response of Filters." *Wireless Engineer*, March 1946, p. 84.

[2] C. C. Eaglesfield. "Transient Response of Filters." *Wireless Engineer*, November 1946, p. 306.

Thermistors at High Frequencies

To the Editor, "Wireless Engineer"

SIR,—I have been using directly-heated high-resistance thermistor* elements to assist impedance measurements on cyclotron models. The models have been operated at about 400 Mc/s and some consideration has been given to the performance of the high-resistance thermistor at these frequencies. Three points have been considered; lead reactance, skin effect and distributed capacitance. Simple calculation has shown that the first two of these cause very small changes in thermistor resistance for the frequency range 0–400 Mc/s. The distributed capacitance does, however, exert considerable influence on the performance of thermistors which have a resistance of over a few thousand ohms.

Using the equivalent transmission line method suggested by Howe[1,2] for carbon-rod resistors, an estimate has been made of the high-frequency resistance of a thermistor of known resistance to direct current. For calculation, the thermistor was regarded as a cylinder of diameter 0.075 cm and length 0.025 cm. These values were taken after measurement with a travelling microscope. With these dimensions the calculated capacitance per unit length of the thermistor becomes about 3.0 $\mu\mu$F.

To calculate the effective impedance we now regard the thermistor as equivalent to a shorted transmission line of length l, resistance per unit length r and capacitance per unit length c where

l = semi-length of thermistor bead
r = resistance per unit length of line; i.e., R/l
R = d.c. resistance of thermistor
c = capacitance per unit length of thermistor = 3.0 $\mu\mu$F per cm in our case.

The input impedance of such a line is quoted by Howe as

$$Z = Z_0 \tanh \alpha l$$

where

$$Z_0 = \sqrt{r/j\omega c}$$

$$\alpha = \sqrt{j\omega c r}$$

For αl large we can take

$$Z \approx Z_0 \approx \sqrt{r/j\omega c}$$

This impedance is equivalent to a resistance $\sqrt{2r/\omega c}$ and a capacitance of reactance $\sqrt{2r/\omega c}$ in parallel.

Thus, for sufficiently high values of resistance and frequency, the high-frequency resistance of the thermistor varies as the square root of the resistance to direct current. This condition is satisfied in our case of resistance greater than 30,000 ohms and frequency of about 400 Mc/s.

Calculated values of the high-frequency resistance for different d.c. resistance values are given in column 2 of the following table.

These can be compared with measured values given in column 3.

D.C. resistance (MΩ)	Calculated h.f. resistance (MΩ)	Measured h.f. resistance (MΩ)
8.9	0.39	0.30
0.073	0.035	0.025

The measured high-frequency resistance has been obtained by using the thermistor as a load on a shorted quarter-wave transmission line. The thermistor was actually soldered between the outer conductor and an insulated core of the inner conductor to enable the 50-c/s control voltage to be applied to the thermistor and to enable d.c. resistance measurements to be made. Screening of the thermistor to prevent radiation was obtained by continuing the outer conductor. The line was formed from brass tube and was damped by a small ring of plasticine attached to the inner conductor to bring the Q down to about 500. From the measured Q and the known dimensions of the line the effective resonant impedance at the open end was calculated. With the thermistor mounted across the open end, the Q of the line was again measured for different thermistor resistances. From the known Q and unloaded input impedance, the effective thermistor resistance was determined from the relation

$$\frac{Q_1}{Q_2} = \frac{R_E + R_i}{R_E}$$

Q_1 = Q without thermistor
Q_2 = Q with thermistor
R_E = Effective resistance of thermistor
R_i = Input resistance of unloaded line at resonance

The possibility of comparing calculated and measured capacitances for the thermistor was

[1] G. W. O. Howe, *Wireless Engineer*, Vol. 12, p. 291, 1935.
[2] G. W. O. Howe, *Wireless Engineer*, Vol. 17, p. 471, 1940.

* The thermistor is a small bead composed of a mixture of metallic oxides and enclosed in a small glass capsule with sealed-in leads. The main characteristic is a very large negative temperature coefficient of resistance. In the directly-heated type of thermistor, the resistance is varied by means of the current through the bead. An indirectly-heated type is also made in which the temperature and, hence, resistance are controlled by a separate heating wire. See G. L. Pearson, *Physical Review*, Vol. 57, p. 1065, 1940.

rejected as the calculated value was of the order 0.001 $\mu\mu$F, which was certainly far too low to measure with the equipment used. It was merely checked that the capacitance was small enough to produce no measurable change in resonant frequency when the effective thermistor impedance changed by a factor of 10. For the apparatus used this meant that the change in capacitance was less than 0.002 $\mu\mu$F.

I am grateful to Standard Telephones and Cables Ltd. for giving the thermistor elements which were specially prepared under the direction of H. Wolfson. Dr. J. H. Fremlin suggested the use of thermistors in cyclotron model measurements and has helped greatly with several stimulating discussions.

Nuffield Physics Laboratory, J. WALKER.
University of Birmingham.

Effect of Aircraft on Fading

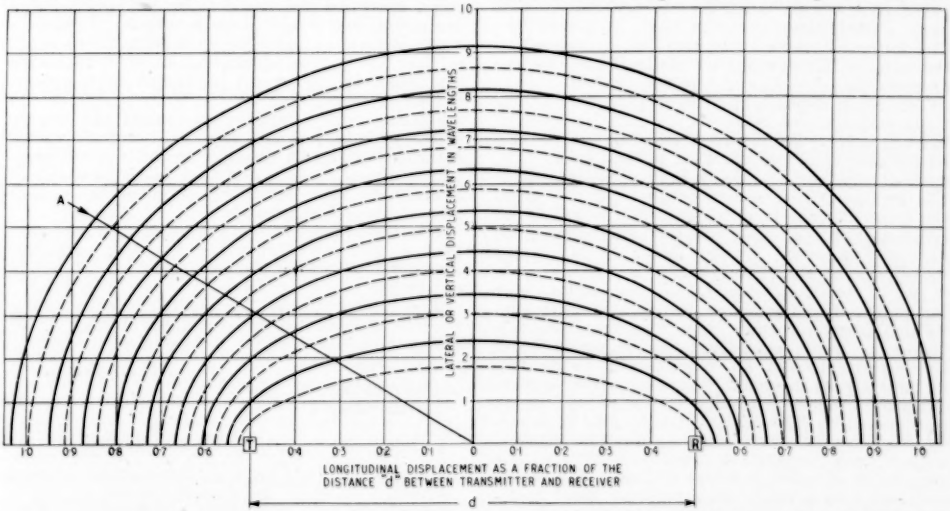
To the Editor, "Wireless Engineer."

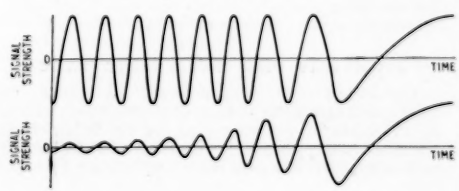
SIR,—While some beam communication trials on 30 to 40 Mc/s and 65 to 100 Mc/s were in progress in 1944, the attention of the writer was brought to frequent occurrences of fading signals in which the fading exhibited the characteristics of steadily changing amplitude and frequency of fade. It was subsequently proved that the phenomenon was due to the presence of aircraft in the transmitted beam, and although the effect was generally most noticeable when field strength measurements were in progress within a mile or two of the transmitter, it was also observed, but to a less extent, when transmitter and receiver were at, or a little beyond, the limits of optical range.

Under the latter conditions of transmitter-receiver separation, there is normally only the direct ray from the transmitter to the receiver; but when there is an aircraft in the vicinity, there may also be at the receiver an indirect ray reflected from the aircraft. This indirect ray will have travelled over a greater distance than the direct ray, and consequently there will be a phase difference between the two signals at the receiver, the resultant being a minimum or maximum according to whether the phase difference is $(2n + 1)\pi$ or $2n\pi$. Thus, as the aircraft position changes, there is a variation of field strength at the receiver.

The accompanying figure shows the positions of an aircraft relative to the transmitter and receiver which will give signal reinforcement (shown by full lines) at the receiver, and also the positions which will result in a signal minimum at the receiver (shown by dotted lines). The loci of these positions, when plotted in a plane passing through the line joining the transmitter and receiver, are a set of confocal ellipses, with the transmitter and receiver as foci. The ellipses have been plotted with distances parallel to the line joining the transmitter and receiver measured in terms of "d," the distance between transmitter and receiver. Lateral displacements from this line are measured in terms of wavelengths.

The resulting variation in field strength due to an aircraft is shown for the particular case of an aircraft flying along the path AO. Curve (a) shows the rate of fading without taking into account





the probable amplitude of the reflected wave. Curve (b) shows the type of fading which might be expected due to the reduction of the reflected signal strength as the aircraft travels further from the transmitter. The latter curve agrees closely with observations which have been made in connection with the alignment of beam transmitters.

Potters Bar, Middlesex. J. W. WHITEHEAD.

(*Correspondence continued overleaf.*)

Spontaneous Fluctuations in a Double-Cathode Valve

To the Editor, "Wireless Engineer"

SIR,—W. Schottky[1] has indicated that the fluctuations of electricity arising in a "valve" with two similar emitting filaments at the same temperature in a state of thermal equilibrium must obey at once both the formula for the "shot effect" as derived by Schottky[2] himself and for thermal noise as derived by H. Nyquist[3]. D. O. North[4] has also discussed this problem; he has shown in detail that, since in such a valve the conductance g (i.e., $\frac{\delta I}{\delta V}$ where I is the mean resultant current throughout the "valve") must obey the formula :—

$$g = \frac{eJ}{kT} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (1)$$

where

e = the magnitude of the electronic charge,
k = Boltzmann's constant (1.37×10^{-23} joules/°K.)
T = the temperature in degrees Kelvin,
J = the mean current traversing the valve, in either direction, assumed identical.

It follows that the current fluctuations should satisfy the relationship :

$$\overline{\triangle I^2} = 4gkT\triangle f \quad .. \quad .. \quad .. \quad .. \quad (2a)$$
$$= 4eJ\triangle f \quad .. \quad .. \quad .. \quad .. \quad (2b)$$

where

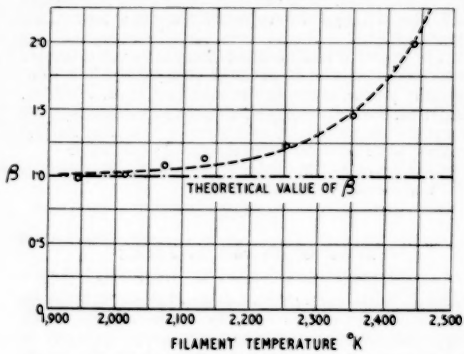
$\overline{\triangle I^2}$ represents the mean square fluctuation of I from its average zero value.
$\triangle f$ is the integrated bandwidth of the measuring instrument.

The latter expression may then be interpreted as the sum of the combined "shot effects" of the currents J.

This result depends on a demonstration that the "space-charge reduction factor" Γ^2, in such a valve is unity. North points out that such should be the case since any variation in the depth of the space-charge barrier arising from a fluctuation in the emission from either cathode should affect the mean current in either direction equally; no compensation of the primary fluctuations, as normally expressed by the factor Γ^2 in normal valves, should therefore arise.

As a result, originally, of discussions with R. Fürth, Edinburgh, on the fundamental connection between "shot" and "thermal" fluctuations, the writer considered that it would be of interest to attempt an experimental verification of this classical theorem. Valves with two tungsten filaments in close proximity—for which the writer is indebted to The M.O. Valve Co., Ltd., Osram Works, London—were employed. An amplifier with a mean response at about 1.6 Mc/s with a bandwidth of a few kc/s was utilized. The results obtained over a range of temperature about 2,000–2,500°K are shown in the figure, where the ordinate β is the ratio of measured "fluctuation temperature" to true temperature.

It appears probable therefrom that the theorem would be obeyed for sufficiently low temperatures; the experimental difficulties, however, associated with diminishing temperatures increase most seriously; the valve dynamic resistance increases rapidly—not only in an absolute sense—but also in comparison with the residual dynamic impedance of the amplifier input itself; further, from equation (2) the magnitude of the generated fluctuations diminishes, and finally it becomes increasingly difficult to guarantee maintenance of equilibrium in the valve. It is hoped later to confirm the behaviour of β at even lower temperatures.



Meanwhile, however the rapid increase of β at higher temperatures which has been carefully confirmed appears of interest. It has often been speculated (e.g., North[5], Moullin[6]) that "excess noise" observed with increasing cathode temperature in normal valves arises from the emission from the cathode of a small amount of positive ions. These, while directly contributing negligible noise, are presumed to influence the space-charge severely during their "leisurely transit." Such an explanation appears to the writer difficult to apply in this case since, according to the discussion mentioned in the second paragraph of this note, the effect of the ions on the space-charge should "balance out." Further, one would presumably expect such ions in any case to be emitted with a Maxwell-Boltzmann distribution at the temperature of the cathode(s) and thus to be incapable of altering the basic result (equation 2a) derived on general statistical grounds.

It is hoped later to carry out a similar investigation on a "valve" with oxide-coated cathodes; considerably lower temperatures can thus be examined.

The writer would like here to acknowledge warmly the invaluable assistance of Mr. E. W. Houghton of the Military College of Science in this investigation.

D. K. C. MACDONALD.
Military College of Science,
Shrivenham.*

1 Schottky, W.: *Z. Phys.*, Vol. 104, p. 248, 1936.
2 Schottky, W.: *Ann. d. Phys.*, Vol. 57, p. 541, 1918.
3 Nyquist, H.: *Phys. Rev.*, Vol. 32, p. 110, 1928.
4 North, D. O.: *R.C.A. Rev.*, Vol. 4, p. 466, 1940.
5 North, D. O.: *R.C.A. Rev.*, Vol. 5, p. 115, 1940.
6 Moullin, E. B.: "Spontaneous Fluctuations of Voltage," Clarendon Press. Cap. 6.

* Now of Clarendon Laboratory, Oxford.

Abstracts and References Index

The Index to the Abstracts and References published in 1946 is in course of preparation and will be available in February, priced 2s. 8d. (including postage). As supplies will be limited our Publishers ask us to stress the need for early application for copies.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

577 969.—Disposition of the carbon-content in a directional microphone of the velocity or pressure type.

H. Dewhurst. Application date 15th February, 1935. (Secret patent: published 7th May, 1946.)

AERIALS AND AERIAL SYSTEMS

577 750.—Flexible mounting for the base of a folded dipole aerial for use on the roof of a road vehicle.

The General Electric Co. Ltd., G. C. L. Campbell, W. Grimbaldeston. Application date 1st March, 1943.

578 018.—Dipole aerial, with coaxial reflectors mounted above and below, for broadcasting centimetre waves outwards and downwards over a restricted service-area.

D. I. Lawson, D. Weighton and Pye Ltd. Application date 8th April, 1943.

578 457.—Dipole aerial, comprising conical and disk-shaped elements, suitable for use on an aeroplane.

Standard Telephones and Cables, Ltd. (assignees of A. G. Kandoian). Convention date (U.S.A.) 15th May, 1943.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

577 453.—D. F. receiver in which the signals from two or more separate aerials are combined to generate an harmonic frequency which varies in amplitude or phase in accordance with the direction of the incoming signals.

J. Robinson. Application dates 19th April, 1939, and 28th March, 1940.

577 455.—Frequency-changing device for use in a direction finder receiving signals from two separate aerials, combined with means for selecting a given harmonic frequency (divided from 577 453).

J. Robinson. Application dates 19th April, 1939, and 28th March, 1940.

577 458.—Directional signalling system in which one or more gas-discharge tubes are utilized to modulate a narrow-central zone in an existing beam of radiation.

H. Hughes and Son Ltd., A. H. W. Beck, and A. J. Hughes. Application date 10th January, 1941.

577 460.—D. F. system wherein a directional aerial is constantly rotated in synchronism with an electromagnet which is controlled by the incoming signals to actuate a pointer.

H. E. Sjöstrand. Convention date (Sweden) 26th March, 1942.

577 527.—Blind-landing system in which one of two overlapping beams is more sharply directive in a vertical plane than the other (addition to 577 276).

Standard Telephones and Cables Ltd., C. W. Earp, and G. G. Samson. Application date 19th September, 1941.

577 672.—Spaced array of aerials, coupled to a common generator through phase-delay networks, for scanning a predetermined area in a radiolocation system.

Hazeltine Corporation (assignees of A. V. Loughren) Convention date (U.S.A.) 26th May, 1941.

577 674.—Radiolocation system in which the beam from a fixed aerial array is swung by electric control so as to scan systematically the area to be explored (divided out of 577 672).

Hazeltine Corporation (assignees of A. V. Loughren), Convention date (U.S.A.) 26th May, 1941.

577 742.—Arrangement for maintaining correct phase and amplitude relations between the signals received on the directional and omni-directional aerials in a sense-determining d.f. receiver.

Marconi's W.T. Co. Ltd., S. B. Smith, and J. F. Hatch. Application date 11th October, 1940.

577 824.—Navigational system in which two spaced transmitters radiate characteristic pulses, so that a mobile craft can follow a predetermined course by maintaining a given time-interval between their reception.

Standard Telephones and Cables, Ltd. (communicated by International Standard Electric Corporation). Application date 14th May, 1942.

577 853.—Blind-landing aerial in which the phase-excitation is arranged to offset distortion due to earth-reflections.

Standard Telephones and Cables, Ltd., and E. H. Ullrich. Application date 25th September, 1940.

577 854.—Radiolocation system in which each exploring pulse partly overlaps the resulting echo, the residual echoes being utilized to give a distance-indication in a graduated milliammeter.

Standard Telephones and Cables, Ltd. (communicated by International Standard Electrical Corporation). Application date 26th December, 1941.

577 855.—Radiolocation system in which auxiliary out-of-phase pulses are utilized to offset undesired reflections from the ground or from fixed objects.

Standard Telephones and Cables, Ltd., and H. G. Busignies. Convention date (U.S.A.) 5th March, 1941.

577 937.—Radiolocation system in which the distance is indicated by the charge developed in a thyratron-controlled capacitor during the interval between the outgoing pulse and the incoming echo.

J. Forman and Pye Ltd. Application date 17th April, 1941.

577 939.—Aerial and reflector system, located at or below ground level, for radiating a navigational beam of rotational symmetry.

H. M. Dowsett. Application date 6th June, 1941.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

577 641.—Broadcast receiver in which the press-button tuning control also brings a rotary frame aerial into orientation with the selected transmitter.
E. K. Cole Ltd., and A. W. Martin. Application dates, 16th March, and 3rd November, 1944.

577 775.—Discriminator circuit, for stabilizing frequency-modulated signals on centimetre waves, comprising two velocity-modulating valves of the resonator type (divided out of 577 758).
E. L. C. White. Application date 25th March, 1944.

577 795.—Remote-control system for use say in radiolocation wherein pulsed signals are transmitted over a radio link to synchronize distant apparatus with the rotation of a local directional aerial.
W. Jones and Pye Ltd. Application date 22nd January, 1943.

577 796.—Remote-control system for aligning a local with a distant rotating indicator, applicable to radiolocation equipment.
L. W. Germany and Pye Ltd. Application date 19th February, 1943.

577 800.—Frequency-discriminator for developing a fixed beat-frequency from a variably-tuned circuit and any one of a series of standard oscillators.
The General Electric Co. Ltd., N. R. Bligh, and J. B. L. Foot. Application date 5th April, 1943.

577 801.—Cathode-follower valve circuit, arranged to give an output which is proportional in magnitude and sense to the difference between two input voltages.
D. G. O. Morris and Metropolitan-Vickers Electrical Co. Ltd. Application date 16th April, 1943.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

577 216.—Preventing the d. c. component of a sawtooth waveform from entering the deflecting coils of a c.r. tube, used say, for television.
Marconi's W.T. Co. Ltd. (assignees of T. T. Eaton). Convention date (U.S.A.) 25th September, 1941.

577 670.—Television system in which, to ensure secrecy, certain of the synchronizing signals are provided by separately-phased local generators at the transmitter and receiver.
Scophony Ltd., and G. Wikkenhauser. Application date, 22nd April, 1938.

577 758.—Stabilizing device for frequency-modulated signals, such as television, where there is no definite or constant mean-frequency.
E. L. C. White. Application date 25th March, 1944.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

577 572.—Combined transmitter-and-receiver installation for carrier-wave signalling over a power-line through a common tuned input-output coupling and a frequency-changing circuit.
Standard Telephones and Cables Ltd. (communicated by International Standard Electric Corporation). Application date 6th November, 1943.

577 710.—Impulse-generator of the multivibrator type comprising means for varying the repetition-frequency, and also the duration of the impulses.
E. A. Newman. Application date 8th March, 1944.

577 713.—Pulse-generator of the blocking-oscillator type in which high-frequency stability is combined with a short build-up time.
Standard Telephones and Cables Ltd. (assignees of A. Alford). Convention date (U.S.A.) 12th January, 1943.

CONSTRUCTION OF ELECTRONIC DISCHARGE DEVICES

577 599.—The use of barium thorate as the activating material in a thermionic cathode surrounded by a perforated metal container.
The General Electric Co. Ltd., S. H. Noble, H. P. Rooksby, W. G. S. Branson, and J. S. Herriott. Application date, 19th January, 1944.

577 602.—Thermionic filament consisting of a plurality of bifilar windings in which the heating current flows in opposite directions through adjacent turns.
Standard Telephones and Cables Ltd (assignees of C. V. Litton). Convention date (U.S.A.) 30th January, 1943.

SUBSIDIARY APPARATUS AND MATERIALS

577 585.—Method of adding a thalliam content to the barrier on boundary layer of a rectifier of the selenium type.
Westinghouse Brake and Signal Co. Ltd., L. E. Thompson, and A. Jenkins. Application date 2nd March, 1944.

577 608.—Band-pass filter in which the mid-band frequency can be varied over a wide range, whilst preserving a constant impedance-ratio.
F. G. Clifford. Application date, 11th February, 1944.

577 613.—Capacitance arrangement for varying the natural frequency of a coaxial resonator, as used for ultra-short waves.
" Patelhold " Patentverwertungs & Elektro-Holding A.G. Convention date (Switzerland), 18th February, 1943.

577 616.—Spraying or sputtering process for preparing the counter-electrode of a rectifier of the selenium type.
Westinghouse Brake and Signal Co., Ltd., L. E. Thompson, and A. Jenkins. Application date, 2nd March, 1944.

577 662. Compensating for temperature-variations in cavity-resonators as used for signalling through waveguides.
Automatic Telephone and Electric Co. and T. H. Turney. Application date, 3rd May, 1944.

577 663.—Circuit which is triggered by a firing pulse of one microsecond to generate a sweep voltage, or an abrupt change of potential, for application to a c. r. indicator.
A. C. Cossor Ltd., B. C. Fleming-Williams, and A. Allen. Application date 4th May, 1944.

577 859.—Piezo-electric crystal with roller electrodes which make contact only at points, or along lines, in the nodal plane of oscillation.
The General Electric Co. Ltd. and E. A. Fielding. Application date, 9th September, 1942.